Department of Mathematics
University of Houston
Numerical Analysis I
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Numerical Analysis II
(5th Homework Assignment)

Exercise 13 (*Affine invariance of Runge Kutta methods*)

Consider the initial value problem

$$(*) \qquad y'(x) \;=\; f(x, y(x)) \quad , \quad y(a) \;=\; \alpha \;,$$

where $f : \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}^m, a \in \mathbb{R}$ and $\alpha \in \mathbb{R}^m$. After the transformation $\hat{y} = Ay$ with regular $A \in \mathbb{R}^{m \times m}$, the initial value problem reads as follows

$$(**) \qquad \hat{y}'(x) \;=\; A\, f(x, A^{-1}\hat{y}(x)) \quad , \quad \hat{y}(a) \;=\; A\, \alpha \;.$$

Show that any Runge Kutta method inherits this property, i.e., is affine invariant in the following sense: If the application of a Runge Kutta method to $(*)$ results in a grid function $y_k, k \in \mathbb{N}_0$, then the application of the same method applied to $(**)$ gives the grid function $Ay_k, k \in \mathbb{N}_0$.

5 Points

Exercise 14 (*Maximal order of consistency of s-stage Runge Kutta methods*)

Show that the application of an s-stage Runge Kutta method to the initial value problem

$$y'(x) \;=\; \lambda\, y(x) \quad , \quad y(x_0) \;=\; y_0$$

yields increment functions of the form

$$k_j(x_k, y_k) \;=\; p_j(\lambda h)\; y_k \quad , \quad 1 \leq j \leq s \;,$$

where p_j are polynomials of degree at most j. Use this result to deduce the maximal order of consistency of s-stage Runge Kutta methods.

5 Points

Exercise 15 (*Runge Kutta method without memory*)

(i) Determine all Runge Kutta methods of order 2 with the Butcher scheme

$c_1 = 0$			
c_2	c_2		
c_3	0	c_3	
	0	0	1

(ii) In algorithmic notation, give an efficient implementation with regard to storage.

(iii) Compute the polynomials p_j , $1 \leq j \leq 3$, introduced in Exercise 14.

6 Points

Delivery of the homework at latest on March 21, 2006. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class.