



University of Houston
Department of Mathematics
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Numerical Analysis II, Spring 2010



Numerical Analysis II (2nd Practical Homework)

Practical homework 2: (*Stetter's method*)

Solve the initial value problem

$$\begin{aligned}y_1'(x) &= y_2(x) \quad , \quad y_1(0) = 30 \quad , \\y_2'(x) &= 0.0032 - 0.4y_2^2(x) \quad , \quad y_2(0) = 0\end{aligned}$$

with the exact solution

$$\begin{aligned}y_1(x) &= 30 + 2.5 \ln(\cosh(\sqrt{0.0128}x)) \quad , \\y_2(x) &= \sqrt{0.08} \tanh(\sqrt{0.0128}x)\end{aligned}$$

on $[0, 10]$ using Stetter's implicit two-step method

$$\begin{aligned}y_{i+1} &= y_{i-1} + h((1-b)f(x_{i-1}, y_{i-1}) + 2bf(x_i, y_i) + \\&\quad + (1-b)f(x_{i+1}, y_{i+1})) \quad , \quad i \in \mathbf{N} \quad , \\y_1 &= y_0 + h(bf(x_0, y_0) + (1-b)f(x_1, y_1))\end{aligned}$$

where $b \in [0, 1]$ is a prespecified parameter.

Generate a code with input given by the parameter b and the number $n \in \mathbf{N}$ of integration steps which computes approximations for fixed step size $h := 10/n$.

The output consists of the errors $e_\nu(x_i) := y_\nu(x_i) - y_{h,\nu,i}$, $1 \leq \nu \leq 2$ at the nodes $x_i := ih$, $1 \leq i \leq n$.

Choose $b = \frac{1}{3}$, $b = \frac{1}{2}$, $b = \frac{2}{3}$ and $n = 20$, $n = 40$, $n = 80$, $n = 100$.

Delivery: Output of the code including comments and tables containing the errors at $x = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ for all given parameter sets.

Control results: Errors for $b = 1/3$ and $h = 0.1$

x	$e_1(x)$	$e_2(x)$
1	$1.9073e - 06$	$2.0042e - 06$
2	$5.7220e - 06$	$3.3304e - 06$
3	$1.1444e - 05$	$3.4347e - 06$
4	$2.0981e - 05$	$3.7327e - 06$
5	$3.0518e - 05$	$3.2187e - 06$
6	$3.6240e - 05$	$2.2501e - 06$
7	$4.3869e - 05$	$1.4007e - 06$
8	$4.9591e - 05$	$4.4703e - 08$
9	$5.5313e - 05$	$4.1723e - 07$
10	$6.1035e - 05$	$1.3411e - 07$

Delivery of the practical work at latest on April 16, 2009. The delivery may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class.