University of Houston
Department of Mathematics
Dr. Ronald H.W. Hoppe
Numerical Analysis II, Spring 2010

Numerical Analysis II (3rd Practical Homework)

Exercise 3: (*Extrapolation method*)

The differential equations, describing the motion of a satellite around the sytem 'earth/moon' with respect to the coordinates $x = (x_1, x_2)$ of a rotating coordinate system, are given by

$$\begin{aligned} x_1'' &= x_1 + 2x_2' - \hat{\mu}\,\frac{x_1+\mu}{N_1} - \mu\,\frac{x_1-\hat{\mu}}{N_2} \ , \\ x_2'' &= x_2 - 2x_1' - \hat{\mu}\,\frac{x_2}{N_1} - \mu\,\frac{x_2}{N_2} \ , \end{aligned}$$

where

$$N_1 := ((x_1+\mu)^2 + x_2^2)^{3/2} \quad , \quad N_2 := ((x_1-\hat{\mu})^2 + x_2^2)^{3/2}$$

and

$$\mu := 0.012277471 \quad , \quad \hat{\mu} := 1 - \mu \ .$$

Here, μ is the ratio of the mass of the moon and the mass of the system. The spatial unit for the Euclidian norm $|x| := (x_1^2 + x_2^2)^{1/2}$ is the average distance earth-moon, whereas the temporal unit is a month. The initial values

$$x_1(0) := 0.994 \ , \ x_1'(0) := 0 \ , \ x_2(0) := 0 \ , \ x_2'(o) := -2.001585106$$

are chosen such that we obtain the clover-leaf like Arenstorf orbit. The period of that orbit is

$$T := 17.0652166 \ .$$

Solve the initial value problem using extrapolation on $[0,T]$ with respect to $N = 100, 200, 400, 800, 1600, 3200, 6400$ macro steps of step size $H := T/N$. As basis schemes use the

- explicit Euler method,

- explicit midpoint rule with an initial Euler step and Gragg's final step.

As step size sequence for the micro steps of the basis schemes use the even harmonic sequence $H/2, H/4, H/6, ..., H/d$ up to a prespecified depth $d \in \mathbb{N}$.

For $d = 2, 4, 6$, the code should provide a protocol file *prot* of the form

$$\begin{array}{ccccc} t_0 & x_1(t_0) & x_1'(t_0) & x_2(t_0) & x_2'(t_0) \\ t_1 & x_1(t_1) & x_1'(t_1) & x_2(t_1) & x_2'(t_1) \\ t_2 & x_1(t_2) & x_1'(t_2) & x_2(t_2) & x_2'(t_2) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ t_N & x_1(t_N) & x_1'(t_N) & x_2(t_N) & x_2'(t_N) \end{array}$$

where N denotes the total number of steps.

Delivery: Output of the code including comments and a visualization of the Arenstorf orbit for $d = 6$ (plot $(x_1(t_i), x_2(t_i)), 0 \le i \le N$).

Control results:

a) Euler method

N	d	$x_1(T)$	$x_1'(T)$	$x_2(T)$	$x_2'(T)$
100	2	2.3867e+01	6.1886e+01	6.0457e+01	-2.0295e+01
1600	4	9.4754e-01	3.9480e-01	5.3820e-01	-2.5865e-01
6400	6	9.9400e-01	2.2916e-03	1.4070e-05	-2.0009e+00

b) Midpoint rule

N	d	$x_1(T)$	$x_1'(T)$	$x_2(T)$	$x_2'(T)$
100	2	2.1761e+01	3.9072e+01	3.7768e+01	-1.9520e+01
1600	4	9.8755e-01	3.2447e-01	3.0334e-01	-2.5454e-01
6400	6	9.9400e-01	-8.1800e-07	-5.0000e-09	-2.0016e+00

Delivery of the practical work at latest on April 16, 2009. The delivery may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class.