



Department of Mathematics  
 University of Houston  
 Numerical Analysis II  
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## Numerical Mathematics II (5th Practical Assignment)

**Exercise 5:** (*Runge Kutta Fehlberg method*)

A seven stage, embedded Runge Kutta Fehlberg method of type RK5(4) has been named after *Dormand* and *Prince*. It is characterized by the Butcher scheme

0							
$\frac{1}{5}$	$\frac{1}{5}$						
$\frac{3}{10}$	$\frac{3}{40}$	$\frac{9}{40}$					
$\frac{4}{5}$	$\frac{44}{45}$	$-\frac{56}{15}$	$\frac{32}{9}$				
$\frac{8}{9}$	$\frac{19372}{6561}$	$-\frac{25360}{2187}$	$\frac{64448}{6561}$	$-\frac{212}{729}$			
1	$\frac{9017}{3168}$	$-\frac{355}{33}$	$\frac{46732}{5247}$	$\frac{49}{176}$	$-\frac{5103}{18656}$		
1	$\frac{35}{384}$	0	$\frac{500}{1113}$	$\frac{125}{192}$	$-\frac{2187}{6784}$	$\frac{11}{84}$	
	$\frac{35}{384}$	0	$\frac{500}{1113}$	$\frac{125}{192}$	$-\frac{2187}{6784}$	$\frac{11}{84}$	0
	$\frac{5179}{57600}$	0	$\frac{7571}{16695}$	$\frac{393}{640}$	$-\frac{92097}{339200}$	$\frac{187}{2100}$	$\frac{1}{40}$

Provide a code which solves the satellite problem from Exercise 4 (4th Practical Assignment) by using the *Dormand/Prince* RKF5(4) method combined

with step size control and Fehlberg's trick to reduce the number of function evaluations.

Choose  $h_0 = 10^{-4}$  as initial step size and

$$TOL = 10^{-10}, 10^{-12}, 10^{-14}, 10^{-16}, 10^{-18}, 10^{-20}$$

as tolerance in the step size control.

The code should provide a protocol file *prot* of the form

$$\begin{array}{ccccc} t_0 & x_1(t_0) & x'_1(t_0) & x_2(t_0) & x'_2(t_0) \\ t_1 & x_1(t_1) & x'_1(t_1) & x_2(t_1) & x'_2(t_1) \\ t_2 & x_1(t_2) & x'_1(t_2) & x_2(t_2) & x'_2(t_2) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ t_N & x_1(t_N) & x'_1(t_N) & x_2(t_N) & x'_2(t_N) \end{array}$$

where  $N$  denotes the total number of steps.

**Delivery:** Output of the code including comments and a graphical visualization of the Arenstorf orbit.

**Control results:**

TOL	N	$x_1(T)$	$x'_1(T)$	$x_2(T)$	$x'_2(T)$
$10^{-10}$	75	-1.3224e-01	-4.0660e-01	-1.1211e+00	-1.4286e-01
$10^{-12}$	118	-1.8399e-01	-4.4155e-01	-1.1352e+00	-8.2852e-02
$10^{-14}$	190	-4.0103e-01	7.3138e-01	-5.5213e-01	1.2752e-01
$10^{-16}$	331	8.8687e-01	5.9272e-01	4.1903e-02	6.3018e-03
$10^{-18}$	524	8.8815e-01	5.9410e-01	4.1914e-02	2.8523e-03
$10^{-20}$	935	9.9400e-01	-2.4169e-02	-1.5334e-04	-2.0013e+00

**Delivery of the practical work at latest on April 13, 2006. The delivery may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class.**