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Numerical Partial Differential Equations

Numerical Partial Differential Equations (Homework 1)

Exercise 1 (*Five-Point Finite Difference Approximation of Poisson's Equation*)

Discretize Poisson's equation with Dirichlet boundary conditions

$$\begin{cases} -\Delta u = f \quad \text{in} \quad \Omega = (0,1) \times (0,1) \\ \qquad u = u^d \quad \text{on} \quad \partial\Omega \end{cases}$$

by the standard five-point difference approximation with respect to a uniform grid of step size h and solve the resulting algebraic system using the Gauss-Seidel iteration, where

$$u(x,y) = e^{xy}, \quad f(x,y) = -(x^2+y^2)e^{xy} \ \text{ in } \ \Omega \ \text{ and } \ u^d(x,y) = e^{xy} \ \text{ on } \ \partial\Omega$$

1. Find $\|u - u_h\|_\infty$, where u_h is the finite difference approximation, for the mesh sizes $h = \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$.

$$\text{(Stopping Criterion: } \frac{\|u_h^{k+1} - u_h^k\|_\infty}{\|u_h^{k+1}\|_\infty} \leq \epsilon, \quad \epsilon = 10^{-12})$$

2. Check whether the error $\|u - u_h\|_\infty$ behaves as $O(h^2)$.
Note: Use double precision.

6 Points

Exercise 1 is due on Jan 28, 2008. The homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class