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Linear Alg Part IV

1a) $\det\begin{pmatrix} 2 & 2 & 0 \\ 2 & 4 & 0 \\ 1 & 1 & 2 \end{pmatrix} = *$

cofactor along 3^{rd} column.

$* = 0(-1)^{1+3}\det\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

$\quad + 0(-1)^{2+3}\det\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$

$\quad + 2(-1)^{3+3}\det\begin{pmatrix} 2 & 2 \\ 2 & 1 \end{pmatrix}$

$= 2(2-4) = \boxed{-4}$

b) $\det\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ ← cofactor along 1^{st} row.

$= 2(-1)^{1+1}\det\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} + 0(-1)^{1+2}\det\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$

$+ 1(-1)^{1+3}\det\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = 2.2 + 1 = \boxed{5}$

(2)

2a) $\det\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{pmatrix}$ ← Cofactor along 1st row

$= 2(-1)^{1+1}\det\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 1 & 3 \end{pmatrix} + (-1)^{1+2}\det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 1 & 3 \end{pmatrix}$

$= 2\cdot 2(9-1) - (9-1)$

$= 3\cdot 8 = \boxed{24}$

b) $\det\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ ← Cofactor along 4th row

$= 3(-1)^{4+4}\det\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 3 \end{pmatrix}$

$= 3\left[3-(1+3)\right] = \boxed{-3}$

c) $\det\begin{pmatrix} 1 & 1 & 2 & 2 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ 2 & 0 & 1 & 3 \end{pmatrix}$

$= (-1)^{1+2}\det\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{pmatrix} + (-1)^{3+2}\det\begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{pmatrix}$

(over)

③

$$=(-1)\left[(9+2+2)-(6+1+6)\right]$$
$$+(-1)\left[(3+4+2)-(4+1+6)\right]$$
$$=-\left[13-13\right]-\left[9-11\right]=\boxed{2}$$

3a) $A=\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$

$$\det(A)=2(-1)^{3+3}(2-4)=-4$$

$$C=\begin{pmatrix} +(2) & -(4) & +(1) \\ -(4) & +(4) & -(0) \\ +(0) & -(0) & +(-2) \end{pmatrix}$$

So $A^{-1}=\frac{1}{\det}C^T=\frac{1}{-4}\begin{pmatrix} 2 & -4 & 0 \\ -4 & 4 & 0 \\ 1 & 0 & -2 \end{pmatrix}$

$$\boxed{=\begin{pmatrix} -\frac{1}{2} & 1 & 0 \\ 1 & -1 & 0 \\ -\frac{1}{4} & 0 & 1/2 \end{pmatrix}}$$

(4)

b) $$A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

$$C = \begin{pmatrix} +(2) & -(4) & +(1) \\ -(-1) & +(3) & -(2) \\ +(-1) & -(-2) & +(2) \end{pmatrix}$$

$$\det(A) = 2 \cdot 2 + 0(-4) + 1 \cdot 1 = 5$$

so $$A^{-1} = \frac{1}{\det(A)} C^T = \frac{1}{5}\begin{pmatrix} 2 & 1 & -1 \\ -4 & 3 & 2 \\ 1 & -2 & 2 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 2/5 & 1/5 & -1/5 \\ -4/5 & 3/5 & 2/5 \\ 1/5 & -2/5 & 2/5 \end{pmatrix}}$$

4a) $$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$x_1 = \frac{1}{-1} \det\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix} = -(3-8) = \boxed{5}$$

$$x_2 = \frac{1}{-1} \det\begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix} = -(4-3) = \boxed{-1}$$

⑤

b) $\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$x_1 = \frac{1}{1}\det\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \boxed{0}$$

$$x_2 = \frac{1}{1}\det\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} = \boxed{1}$$

5 a) $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$$\det(A) = 2(2-4) = -4$$

$$x_1 = -\frac{1}{4}\det\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix} = -\frac{1}{4}2(1) = \boxed{-\frac{1}{2}}$$

$$x_2 = -\frac{1}{4}\det\begin{pmatrix} 2 & 1 & 0 \\ 2 & 0 & 0 \\ 1 & 1 & 2 \end{pmatrix} = -\frac{1}{4}2(-2) = \boxed{1}$$

$$x_3 = -\frac{1}{4}\det\begin{pmatrix} 2 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = -\frac{1}{4}\left[(2+2) - (1+4)\right]$$

$$= \boxed{\frac{1}{4}}$$

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5b) $$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\det(A) = (8) - (2+2) = 4$$

$$x_1 = \frac{1}{4}\det\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix} = \frac{1}{4}(3) = \boxed{\frac{3}{4}}$$

$$x_2 = \frac{1}{4}\det\begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix} = \frac{1}{4}(-1)(2) = \boxed{-\frac{1}{2}}$$

$$x_3 = \frac{1}{4}\det\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \frac{1}{4}(1) = \boxed{\frac{1}{4}}$$

6a) $$\det\begin{pmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{pmatrix} = \lambda(\lambda-3) + 2 = \lambda^2 - 3\lambda + 2 = 0$$

Use quad formula

$$\lambda = \frac{3 \pm \sqrt{9-8}}{2} = \left\{\boxed{\begin{matrix} 1 \\ 2 \end{matrix}}\right.$$

b) $$\det\begin{pmatrix} -\lambda & 2 \\ -1 & 3-\lambda \end{pmatrix} = \lambda(\lambda-3) + 2 = \lambda^2 - 3\lambda + 2 = 0$$

(over)

(7)

Quad formula $\lambda = \frac{3 \pm \sqrt{9-8}}{2} = \boxed{\begin{matrix}1\\2\end{matrix}}$

c) $\det\begin{pmatrix}1-\lambda & 2\\ -1 & 4-\lambda\end{pmatrix} = (\lambda-1)(\lambda-4)+2$

$= \lambda^2 - 5\lambda + 6 = 0$

Quad formula $\lambda = \frac{5 \pm \sqrt{25-24}}{2} = \boxed{\begin{matrix}2\\3\end{matrix}}$

d) $\det\begin{pmatrix}-2-\lambda & -2\\ 6 & 5-\lambda\end{pmatrix} = (\lambda+2)(\lambda-5)+12$

$= \lambda^2 - 3\lambda + 2 = 0$

Quad formula $\lambda = \frac{3 \pm \sqrt{9-8}}{2} = \boxed{\begin{matrix}1\\2\end{matrix}}$

7a) $\det\begin{pmatrix}1-\lambda & 2\\ 2 & 2-\lambda\end{pmatrix} = (\lambda-1)(\lambda-2)-4$

$= \lambda^2 - 3\lambda - 2$

Quad formula $\lambda = \frac{3 \pm \sqrt{9+8}}{2} = \boxed{\frac{3 \pm \sqrt{17}}{2}}$

b) $\det\begin{pmatrix}1-\lambda & 1\\ -1 & 1-\lambda\end{pmatrix} = (\lambda-1)^2+1 = 0$

$\boxed{\lambda = 1 \pm i}$

⑧

c) $\det\begin{pmatrix} -\lambda & -1 & -1 \\ 2 & 3-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{pmatrix}$

$= (3-\lambda)\det\begin{pmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{pmatrix}$

$= (3-\lambda)\left(\lambda(\lambda-3)+2\right)$

$= (3-\lambda)\left(\lambda^2-3\lambda+2\right) = 0$

$\lambda = 3$ & Quad formula

$\lambda = \frac{3 \pm \sqrt{9-8}}{2} = 1, 2$

So $\boxed{\lambda = 1, 2, 3}$

d) $\det\begin{pmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & -2 \\ 2 & 2 & 4-\lambda \end{pmatrix}$

criss cross

$= \left((1-\lambda)^2(4-\lambda)\right) - \left(2(1-\lambda) - 4(1-\lambda)\right)$

$= (1-\lambda)^2(4-\lambda) + 2(1-\lambda)$ (over)

⑨

$$=(1-\lambda)\left((1-\lambda)(4-\lambda)+2\right)$$

$$=(1-\lambda)(\lambda^2-5\lambda-6)=0$$

$\lambda=1$ & Quad formula

$$\lambda=\frac{5\pm\sqrt{25-24}}{2}=\begin{cases}2\\3\end{cases}$$

so $\boxed{\lambda=1,2,3}$