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2318 E1
Sp24

1) Compute

a) $D+E = \begin{pmatrix} 1 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 4 \end{pmatrix}$

$\boxed{= \begin{pmatrix} 4 & 6 \end{pmatrix}}$

b) $A+2C = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$

$= \boxed{\begin{pmatrix} 5 & 2 & 3 \\ 1 & 4 & 3 \end{pmatrix}}$

c) $AB = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{pmatrix}$

$= \boxed{\begin{pmatrix} 8 & 9 \\ 8 & 7 \end{pmatrix}}$

d) $DC = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$\boxed{= \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}}$

② 5

2 Write in aug form use GE to solve

a) $\left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 8 \\ 1 & 1 & 1 & 2 & 7 \\ 2 & 1 & 1 & 1 & 8 \end{array}\right]$ Pivot row

$R2 \to R1 - R2$

$R3 \to 2R1 - R3$

$\sim \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 8 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 3 & 1 & 3 & 8 \end{array}\right]$ Pivot row

$R3 = R3 - 3R2$

$\sim \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 8 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 & 5 \end{array}\right]$

Back sub: $\boxed{x_4 = \alpha}$ (free var)

$x_3 + 3\alpha = 5$ $\boxed{x_3 = 5 - 3\alpha}$

$\boxed{x_2 = 1}$

$x_1 + 2 \cdot 1 + 1(5 - 3\alpha) + 2\alpha = 8$

$x_1 = 1 + \alpha$

$$\boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1+\alpha \\ 1 \\ 5-3\alpha \\ \alpha \end{pmatrix}} = \begin{pmatrix} 1 \\ 1 \\ 5 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \end{pmatrix}$$

(3)

b)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 0 & 1 & 3 \\ 3 & 1 & 1 & 4 \\ 3 & 1 & 2 & 5 \end{array}\right]$$

Pivot
$R2 = 2R1 - R2$
$R3 = 3R1 - R3$
$R4 = 3R1 - R4$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 2 & 1 & 1 \end{array}\right]$$

Pivot
$R3 = R3 - R2$
$R4 = R4 - R2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Now it's in echelon form

← No problem here

Back sub?

$X_3 = 1$

$2X_2 + 1 = 1 \quad X_2 = 0$

$X_1 + 1 \cdot 0 + 1 \cdot 1 = 2$

$X_1 = 1$

$$\boxed{\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}$$

④

3a $\begin{pmatrix}5\\6\\7\end{pmatrix} \overset{?}{\in} \operatorname{Span}\left\{\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}3\\2\\1\end{pmatrix}\right\}$

$$\alpha_1\begin{pmatrix}1\\2\\3\end{pmatrix} + \alpha_2\begin{pmatrix}3\\2\\1\end{pmatrix} = \begin{pmatrix}5\\6\\7\end{pmatrix}$$ solvable?

$$\begin{pmatrix}1 & 3\\2 & 2\\3 & 1\end{pmatrix}\begin{pmatrix}\alpha_1\\\alpha_2\end{pmatrix} = \begin{pmatrix}5\\6\\7\end{pmatrix}$$

$$\begin{array}{ccc} 3 & 9 & 15 \end{array}$$

$$\left[\begin{array}{cc|c}1 & 3 & 5\\2 & 2 & 6\\3 & 1 & 7\end{array}\right] \sim \left[\begin{array}{cc|c}1 & 3 & 5\\0 & 4 & 4\\0 & 8 & 8\end{array}\right]$$

$$\sim \left[\begin{array}{cc|c}1 & 3 & 5\\0 & 1 & 1\\0 & 0 & 0\end{array}\right] \Rightarrow \begin{array}{l}\alpha_2 = 1\\ \alpha_1 + 3\cdot 1 = 5\\ \alpha_1 = 2\end{array}$$

Yes

$$\begin{pmatrix}5\\6\\7\end{pmatrix} = 2\begin{pmatrix}1\\2\\3\end{pmatrix} + 1\begin{pmatrix}3\\2\\1\end{pmatrix} \checkmark$$

① $\begin{pmatrix}1\\1\\0\end{pmatrix} \overset{?}{\in} \operatorname{span}\left\{\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}3\\2\\1\end{pmatrix}\right\}$

$$\begin{pmatrix}1 & 3\\2 & 2\\3 & 1\end{pmatrix}\begin{pmatrix}\alpha_1\\\alpha_2\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix}$$ is this solvable?

$$\left[\begin{array}{cc|c}1 & 3 & 1\\2 & 2 & 1\\3 & 1 & 0\end{array}\right] \sim \left[\begin{array}{cc|c}1 & 3 & 1\\0 & 4 & 1\\0 & 8 & 0\end{array}\right]$$

$$\sim \left[\begin{array}{cc|c}1 & 3 & 1\\0 & 1 & 0\\0 & 0 & 1\end{array}\right] \longleftarrow \text{impossible!}$$

NOT solvable

So $\begin{pmatrix}1\\1\\0\end{pmatrix}$ $\underline{\underline{\text{NOT}}}$ in $\operatorname{span}\left\{\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}3\\2\\1\end{pmatrix}\right\}$

⑥

4a) $V_a = \text{span}\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\2\\3\end{pmatrix}\right\}$

To get std basis reduce

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\end{bmatrix}$$ to canonical form

$$\sim \begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 2\end{bmatrix} \sim \begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 2\end{bmatrix}$$

$$\boxed{V_a = \text{span}\underbrace{\left\{\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\1\\2\end{pmatrix}\right\}}_{\text{std basis for } V_a}}$$

$$\boxed{\dim(V_a) = 2}$$

4b $V_b = \text{span}\left\{ \begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}2\\1\\2\end{pmatrix}, \begin{pmatrix}1\\-1\\1\end{pmatrix} \right\}$

to get std basis

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & 3 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{V_b = \text{span}\left\{ \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix} \right\}}$$

std basis

$$\boxed{\dim(V_b) = 2}$$

⑧

5 Null space & std basis for range

a) $\mathcal{L}(\vec{x}) \equiv \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A\vec{x}$

Null Space $A\vec{x} = \vec{0}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 4 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Two free variables

$x_1 + 2\alpha + \beta = 0$
$x_1 = -2\alpha - \beta$ $\Leftarrow$ $x_2 = \alpha$
$x_3 = \beta$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2\alpha - \beta \\ \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

A basis for $N(\mathcal{L})$ is

$$\boxed{\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}}$$

(9)

$$\text{Range}(\mathcal{L}) = \text{span}\left\{ \begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}2\\4\\2\end{pmatrix}, \begin{pmatrix}1\\2\end{pmatrix} \right\}$$

To get std-basis

$$\begin{bmatrix}1 & 2\\2 & 4\\1 & 2\end{bmatrix} \sim \begin{bmatrix}1 & 2\\0 & 0\\0 & 0\end{bmatrix}$$

std- basis for Range is $\boxed{\left\{\begin{pmatrix}1\\2\end{pmatrix}\right\}}$

b) $\mathcal{L}(\vec{x}) = \begin{pmatrix}1 & 0 & 1\\1 & 2 & 3\\2 & 2 & 4\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = A\vec{x}$

Null Space:

$$\left[\begin{array}{ccc|c}1 & 0 & 1 & 0\\1 & 2 & 3 & 0\\2 & 2 & 4 & 0\end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c}1 & 0 & 1 & 0\\0 & 2 & 2 & 0\\0 & 2 & 2 & 0\end{array}\right] \sim \left[\begin{array}{ccc|c}1 & 0 & 1 & 0\\0 & 1 & 1 & 0\\0 & 0 & 0 & 0\end{array}\right]$$

$x_1 = -\alpha \Leftarrow x_2 = -\alpha \Leftarrow$ Free is $x_3 = \alpha$

(10)

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\alpha \\ -\alpha \\ \alpha \end{pmatrix} = \alpha \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\boxed{Null(\mathcal{L}) = Span\left\{ \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\}}$$

This is a basis set for $N(\mathcal{L})$

$$Range(\mathcal{L}) = Span\left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} \right\}$$

To sel std-basis

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 1 & 3 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\text{std-basis for range is } \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}}$$