Math 3331 Exam 1. Sanders Spring 2024

This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. We've considered four basic classes of first order ODE's we can in principle solve. They are: (1) Separable, (2) Linear, (3) Homogeneous, (4) Exact. Classify each of the following but do not solve. An equation may be in more than one class (and you must list all classes if so), or it may be in none. No guessing – show your work!

(a) $x\,\dfrac{du}{dx} + e^u = 0$ (c) $\dfrac{du}{dx} + u + x^2 = 0$

(b) $u^2\,\dfrac{du}{dx} + xu + x^2 = 0$ (d) $(u + x + 1)\,\dfrac{du}{dx} + u + 2 = 0$

2. Find the explicit form general solution to the following.

(a) $\dfrac{du}{dx} = x^2(u^2 + 1)$ (b) $xu\,\dfrac{du}{dx} = u^2 + x^2$

3. Solve the following. If convenient, you may leave the solution in implicit form here.

(a) $\dfrac{du}{dx} + u^2 = 0$ (b) $(u + x)\,\dfrac{du}{dx} + u + x^2 = 0$

4. Recall the cylindrical water tank with a small drain hole given on your applications homework. There we derived the following ODE from Bernoulli's principle

$$\frac{dz}{dt} = -k\sqrt{z},$$

where $z(t)$ is the depth of the water at time t and k is a constant. Suppose initially the water depth is $z(0) = 4$. Also, suppose the water depth is measured when $t = 1$ and found to be $z(1) = 1$.

(a) Use the ODE and supplied data to determine the constant k.
(The answer is $k = 2$. You may freely use this in part b below.)

(b) Determine the time t_* it takes for the tank to empty, that is $z(t_*) = 0$.

5. Find the general solution of the following second order differential equations by using the given factorization.

(a) $$\left(\frac{d}{dx} - I\right)\left(\frac{du}{dx} - u\right) = \frac{d^2u}{dx^2} - 2\,\frac{du}{dx} + u = 0.$$

(b) $$\left(\frac{d}{dx} - I\right)\left(\frac{du}{dx} + u\right) = \frac{d^2u}{dx^2} - u = 1.$$