Math 3338, Fa09: Homework 4

1. Suppose $X \sim B(n,p)$ with $p \in [0,1]$ and $n \in \{0,1,2,\ldots\}$. Derive the following.

(a) $\mu_X = np$ (b) $\sigma_X^2 = np(1-p)$

Hint for (a): The binomial theorem implies $1 = (p+1-p)^m = \sum_{k=0}^m \binom{m}{k} p^k(1-p)^{m-k}$. Use this and the fact that when $n \geq 1$

$$\binom{n}{k} p^k(1-p)^{n-k} \cdot k = np \cdot \frac{(n-1)!}{(k-1)!(n-1-(k-1))!} p^{k-1}(1-p)^{n-1-(k-1)}.$$

Hint for (b): Write

$$\sigma_X^2 + \mu_X^2 = \sum_{k=0}^{n} \binom{n}{k} p^k(1-p)^{n-k}k^2,$$

and calculate that, as done in (a), the right hand side is equal to $np((n-1)p+1)$ when $n \geq 1$.

2. Suppose $X \sim Pos(\lambda)$. Derive the following.

(a) $\mu_X = \lambda$ (b) $\sigma_X^2 = \lambda$

Hint: $\dfrac{\lambda^k}{k!}k = \lambda \dfrac{\lambda^{k-1}}{(k-1)!}$.

3. Suppose $X \sim L(\mu, b)$ with $b > 0$. Derive the following.

(a) $\mu_X = \mu$ (b) $\sigma_X^2 = 2b^2$

4. Suppose $X \sim \chi_\mu^2$ with $\mu > 0$. Derive the following.

(a) $\mu_X = \mu$ (b) $\sigma_X^2 = 2\mu$

Hint: For any $p > 0$ integration by parts gives $\int_0^\infty t^p e^{-t} dt = p \int_0^\infty t^{p-1} e^{-t} dt$.

5. Suppose $X \sim Cauchy(x_0, \gamma)$ with $\gamma > 0$. Show the following.

(a) μ_X is undefined. (b) σ_X^2 is undefined.

Hint: For any $p \geq 1$ show that $\displaystyle\lim_{R\to\infty} \int_0^R \frac{x^p}{1+x^2}\, dx = \infty$.