Some Additional Well-Known Distributions

- Binomial Distribution; [1]. $X \sim B(n,p)$

$$P(X \le x) = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} H(x-k),$$

where $H(x)$ is the Heaviside function and $p \in [0,1]$ and $n \in \{0,1,2,\ldots\}$ are parameters. Recall that the binomial coefficient and H are given by

$$\binom{n}{k} \equiv \frac{n!}{k!\,(n-k)!}, \qquad H(x) \equiv \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \ge 0. \end{cases}$$

- Poisson Distribution; [2]. $X \sim Pois(\lambda)$

$$P(X \le x) = e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} H(x-k),$$

where $\lambda \in \mathbb{R}$ is a parameter.

- Laplace Distribution; [3]. $X \sim L(\mu, b)$

$$P(X \le x) = \begin{cases} \frac{1}{2} \exp\left((x-\mu)/b\right) & \text{if } x < \mu \\ 1 - \frac{1}{2} \exp\left(-(x-\mu)/b\right) & \text{if } x \ge \mu, \end{cases}$$

where $\mu \in \mathbb{R}$ and $b > 0$ are parameters.

- Chi-Square Distribution; [4]. $X \sim \chi^2_\mu$

$$P(X \le x) = \begin{cases} 0 & \text{if } x < 0 \\ \gamma(\mu/2, x/2)/\Gamma(\mu/2) & \text{if } x \ge 0, \end{cases}$$

where $\mu > 0$ is a parameter and where $\Gamma(s)$ denotes the Gamma function and $\gamma(s,y)$ denotes the lower incomplete Gamma function which are defined for $s > 0$ by

$$\Gamma(s) \equiv \int_0^\infty t^{s-1} e^{-t} dt, \qquad \gamma(s,y) \equiv \int_0^y t^{s-1} e^{-t} dt.$$

- Cauchy Distribution; [5]. $X \sim Cauchy(x_0, \gamma)$

$$P(X \le x) = \frac{1}{\pi} \arctan((x - x_0)/\gamma) + \frac{1}{2},$$

where $x_0 \in \mathbb{R}$ and $\gamma > 0$ are parameters.

[1] http://en.wikipedia.org/wiki/Binomial_distribution
[2] http://en.wikipedia.org/wiki/Poisson_distribution
[3] http://en.wikipedia.org/wiki/Laplace_distribution
[4] http://en.wikipedia.org/wiki/Chi-square_distribution
[5] http://en.wikipedia.org/wiki/Cauchy_distribution

- Gamma Distribution; [6]. $X \sim \Gamma(\alpha, \theta)$

$$P(X \le x) = \begin{cases} 0 & \text{if } x < 0 \\ \gamma(\alpha, x/\theta)/\Gamma(\alpha) & \text{if } x \ge 0, \end{cases}$$

where $\alpha > 0$ and $\theta > 0$ are parameters.

- Exponential Distribution; [7]. $X \sim Exp(\lambda)$

$$P(X \le x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & \text{if } x \ge 0, \end{cases}$$

where $\lambda > 0$ is a parameter.

You should be most familiar with the uniform distribution, $U(a, b)$, and the normal distribution, $N(\mu, \sigma^2)$, discussed on Homework 2.

[6] `http://en.wikipedia.org/wiki/Gamma_distribution`
[7] `http://en.wikipedia.org/wiki/Exponential_distribution`