MATH 3363 MIDTERM EXAM I. Sanders Fall '02

This exam has 5 problems and all 5 problems will be graded. You have the full hour and a half to complete it. Use my supplied paper only and return your solution sheets with the problems in order. Put your name, **last name first**, and **social security number** on each solution sheet you turn in. Good luck.

1. Solve the following boundary value problems:

$$\text{(a)} \qquad \frac{d^2}{dx^2}y + y = 0, \qquad y(0) = 0 \quad y(1) = 1.$$

$$\text{(b)} \qquad \frac{d^2}{dx^2}y - y = 0, \qquad y(0) = 0 \quad y(1) = 1.$$

2. The 2π–periodic Fourier series has the form $a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$.

(a) Find the 2π–periodic Fourier series to $\cos^2(x)$.

(b) Find the 2π–periodic Fourier series to x^2. (Answer: $x^2 \sim \dfrac{\pi^2}{3} + 4\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(nx)$.)

(c) Use Parseval and the result in (b) to compute the value of $\sum_{n=1}^{\infty} \frac{1}{n^4}$.

3. By using the *integration by parts* technique, show the following eigenvalue problems have **real eigenvalues**, and associated to distinct eigenvalues, their **eigenfunctions are orthogonal** in the given inner product.

$$\text{(a)} \qquad \frac{d^2}{dx^2}u = \lambda u, \qquad u(0) = 0 \quad u(1) = 0, \qquad (u, v) = \int_0^1 u\, v\, dx.$$

$$\text{(b)} \qquad \frac{d^2}{dx^2}u = \lambda u, \qquad u(-\pi) = u(\pi) \quad u_x(-\pi) = u_x(\pi), \qquad (u, v) = \int_{-\pi}^{\pi} u\, v\, dx.$$

4. Determine all eigenvalues and eigenfunctions to the two eigenvalue problems given in question 3.

5. Consider the *complex valued inner product* $(u, v) = \displaystyle\int_{-\pi}^{\pi} u\, \overline{v}\, dx$.

(a) Show by explicit calculation that the functions $\phi_n(x) = e^{inx}$ with integer n form an orthogonal set.

(b) Given that $f(x) \sim \displaystyle\sum_{n=-\infty}^{\infty} a_n\, \phi_n(x)$, determine a formula for the Fourier coefficients a_n involving an integral of f.

(c) Determine the complex Fourier series for the function $f(x) = x$.