MATH 3363 MIDTERM EXAM I. Sanders Fall '03

This exam has 5 problems and all 5 problems will be graded. You have the full hour and a half to complete it. Use my supplied paper only and return your solution sheets with the problems in order. Put your name, **last name first**, and **social security number** on each solution sheet you turn in. Good luck.

1. Solve the following eigenvalue problems to determine **all** real eigenvalues and associated eigenfunctions. (You may assume λ is real, but nothing more.)

(a) $\dfrac{d^2u}{dx^2} = \lambda u$, $u(0) = 0,\ u(1) = 0$ (b) $\dfrac{d^2u}{dx^2} = \lambda u$, $u_x(0) = 0,\ u_x(1) = 0$

2. By integration, show each given set of functions form an orthogonal set with respect to the inner product $(f, g) = \int_0^1 f(x)g(x)\,dx$.

(a) $\{\sin(n\pi x) : n = 1, 2, 3, \ldots\}$ (b) $\{\cos(n\pi x) : n = 0, 1, 2, \ldots\}$

3. Solve the heat equation $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ on $0 \le x \le 1,\ 0 \le t$ with the given boundary/initial conditions.

(a) $u(0,t) = 0,\ u(1,t) = 0$, $u(x,0) = \sin(\pi x) + 2\sin(2\pi x)$ (b) $u_x(0,t) = 0,\ u_x(1,t) = 0$, $u(x,0) = 1 + \cos(\pi x)$

4. Solve Laplace's equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ on the unit box $0 \le x \le 1,\ 0 \le y \le 1$ with the given boundary conditions.

(a) $u_x(0,y) = 0$, $u_x(1,y) = 0$, $u(x,0) = 1$, $u(x,1) = 0$ (b) $u_x(0,y) = 0$, $u_x(1,y) = \sin(\pi y)$, $u(x,0) = 0$, $u(x,1) = 0$

5a. Expand the function $f(x) = 1$ into a Fourier series in terms of the basis given in **problem 2a** above.

5b. Expand the function $f(x) = \sin^2(3\pi x)$ into a Fourier series in terms of the basis given in **problem 2b** above.