MATH 3363 MIDTERM EXAM I. Sanders Fall 2004

This exam has 5 problems and all 5 problems will be graded. You have one hour to complete it. Use my supplied paper only and return your solution sheets with the problems in order. Put your name, **last name first**, and **social security number** on each solution sheet you turn in. Good luck.

1. Solve the following initial and boundary value problem.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0 \quad \text{when} \quad \begin{array}{ll} \text{(a)} & y(0) = 0 \;\; y_x(0) = -2. \\ \text{(b)} & y(0) = 1 \;\; y(1) = 1. \end{array}$$

2. Determine all eigenvalues and eigenfunctions to the following.

$$\frac{d^2}{dx^2}u = \lambda u, \quad u(0) = 0 \;\; u(1) = 0.$$

You may assume all eigenvalues are real.

3. Our Fourier sine series has the form $\sum_{n=1}^{\infty} a_n \sin(n\pi x)$.

(a) Show $\{\sin(n\pi x)\}_{n=1}^{\infty}$ forms an orthogonal set with respect to $(f, g) = \int_0^1 f(x)g(x)\,dx$.

(b) Compute the sine series for $f(x) = x$. (Answer: $x \sim \sum_{n=1}^{\infty} -2\frac{(-1)^n}{n\pi} \sin(n\pi x)$.)

(c) Use Parseval and the result in (b) to compute the value of $\sum_{n=1}^{\infty} 1/n^2$.

4. Solve the heat equation $u_t = u_{xx}$ on $0 \le x \le 1$, $0 \le t$ with boundary & initial conditions: $u(0,t) = 0$, $u(1,t) = 0$ & $u(x,0) = 2\sin(\pi x) + 5\sin(4\pi x)$.

5. Solve the wave equation $u_{tt} = u_{xx}$ on $0 \le x \le 1$, $0 \le t$ with boundary & initial conditions: $u(0,t) = 0$, $u(1,t) = 0$ & $u(x,0) = 2\sin(\pi x)$, $u_t(x,0) = 3\sin(\pi x)$.