MATH 3363 MIDTERM EXAM I. Sanders Fall 2005

This exam has 5 problems, and all 5 problems will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **social security number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. Solve the following *initial-value* and *boundary-value* problems.

(a) $$\frac{d^2y}{dx^2} + y = 0, \quad y(0) = 1,\ y'(0) = 0$$

(b) $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0, \quad y(0) = 0,\ y(1) = 1$$

2. Consider the eigenvalue problem

$$\frac{d^2u}{dx^2} = \lambda u, \quad u(0) = 0,\ u(1) = 0$$

, and inner product $(u, v) = \int_0^1 u(x)v(x)\,dx.$

Do the next two parts by using the *integration by parts* technique.
(a) Show all eigenvalues must be real numbers.
(b) Show all eigenfunctions associated to distinct eigenvalues must be orthogonal wrt the given inner product.

3(a) (15 points) Determine all eigenvalues and eigenfunctions to the eigenvalue problem given in problem 2 above. They must be correctly enumerated. (In your solution, you may assume λ is real but nothing more.)

3(b) (5 points) By directly calculating integrals, show $\{\sin(n\pi x)\}_{n=1}^{\infty}$ forms an orthogonal set wrt the inner product given in problem 2.

4. Determine the Fourier–sine series, (i.e. $f(x) = \sum_{n=1}^{\infty} a_n \sin(n\pi x)$), for the following.

(a) $f(x) = \sin(\pi x)\cos(\pi x)$ (b) $f(x) = x$

5. Solve the *heat equation*

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{on } 0 < x < 1,\ t > 0,$$

with boundary conditions $u(0, t) = 0$, $u(1, t) = 0$ and initial condition $u(x, 0) = f(x)$ when

(a) $f(x) = \sin(2\pi x) + 3\sin(3\pi x)$ (b) $f(x) = x$