MATH 3363 MIDTERM EXAM I. Sanders Fall 2006

This exam has 5 problems, and all 5 problems will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **social security number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless otherwise indicated.

1(a) Use integration by parts to derive the following formula.

$$\int_a^b u''(x)v(x)\,dx = u'(x)v(x)|_a^b - \int_a^b u'(x)v'(x)\,dx.$$

1(b) Use part (a) to prove all eigenvalues to $\dfrac{d^2u}{dx^2} = \lambda u$ with boundary conditions $u'(0) = 0,\ u'(1) = 0$ are real numbers.

2. Determine whether or not the following are linear operators.

(a) $\mathcal{L}(u) = \dfrac{du}{dx} + u.$ (c) $\mathcal{L}(u) = u\dfrac{du}{dx}.$

(b) $\mathcal{L}(u) = \dfrac{du}{dx} + 1.$ (d) $\mathcal{L}(u) = x^2\dfrac{d^2u}{dx^2} + e^x u.$

3. Solve the *initial-value* and *boundary-value* problems.

(a) $\dfrac{d^2y}{dx^2} + 4y = 0$, $y(0) = 1,\ y'(0) = 0$

(b) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = 0$, $y(0) = 0,\ y(1) = 1$

4(a) (15 points) Determine all eigenvalues and eigenfunctions to

$$\frac{d^2u}{dx^2} = \lambda u, \quad u'(0) = 0,\ u'(1) = 0.$$

These must be correctly enumerated. (Assume λ is real but nothing more.)

4(b) (5 points) By directly calculating integrals, show $\{\cos(n\pi x)\}_{n=0}^{\infty}$ forms an orthogonal set wrt the inner product $(f, g) = \int_0^1 f(x)g(x)\,dx$.

5(a) (10 points) Derive the formulae for the coefficients a_n for the Fourier cosine series $f(x) = \sum_{n=0}^{\infty} a_n \cos(n\pi x)$. (Answer $a_0 = \int_0^1 f(x)\,dx$ and $a_n = 2\int_0^1 f(x)\cos(n\pi x)dx$ for $n = 1, 2, 3, \ldots$.)

Now, calculate the cosine series for the following.

(b) $f(x) = x$ (c) $f(x) = \cos^2(\pi x) - \sin^2(\pi x)$