

Q1.1

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$u(0,t) = 0 \quad u_x(1,t) = 0$$

$$u(x,0) = 9 + 43 \cos(12\pi x)$$

$$\frac{d^2 e}{dx^2} = \lambda e$$

$$e'(0) = 0 \quad e'(1) = 0$$

$$\lambda_0 = 0 \quad e_0(x) = 1 \quad \text{all eigenfunctions s'vals}$$

$$\lambda_n = -(n\pi)^2 \quad e_n(x) = \cos(n\pi x) \quad n = 1, 2, \dots$$

$$u(x,t) = \sum_{n=0}^{\infty} \alpha_n(t) \cos(n\pi x)$$

$$\sum_{n=0}^{\infty} \left( \frac{\partial \alpha_n}{\partial t} \right) \cos(n\pi x) = \sum_{n=0}^{\infty} \left( -(n\pi)^2 \alpha_n \right) \cos(n\pi x)$$

LHS  $\frac{\partial \alpha_n}{\partial t} = -(n\pi)^2 \alpha_n$   $\alpha_n(t) = C_n e^{-(n\pi)^2 t}$

Suppose I know the soln to Q2.1

$$\begin{aligned}
 & \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \quad -\infty < x < \infty \\
 & u(x, 0) = f(x) \quad u_t(x, 0) = 0 \\
 & u(x, t) = \frac{1}{2}(f(x+t) + f(x-t))
 \end{aligned}$$

Can I use above knowledge to solve

$$\begin{aligned}
 & \frac{\partial^2 u}{\partial t^2} - 12 \frac{\partial^2 u}{\partial x^2} = 0 \\
 & u(x, 0) = f(x) \quad ? \\
 & u_t(x, 0) = 0
 \end{aligned}$$

Yes by changing variables

$$x = \sqrt{12} y$$

$$\frac{\partial}{\partial x} = \frac{1}{\sqrt{12}} \frac{\partial}{\partial y}$$

$$\frac{\partial}{\partial x} = \frac{\partial y}{\partial x} \frac{\partial}{\partial y} = \frac{1}{\sqrt{12}} \frac{\partial}{\partial y}$$

$$\frac{\partial^2}{\partial x^2} = \frac{1}{12} \frac{\partial^2}{\partial y^2}$$

$$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{12}} \frac{\partial}{\partial y} \right) = \frac{1}{\sqrt{12}} \frac{1}{\sqrt{12}} \frac{\partial^2}{\partial y^2}$$

$$u(x,t) = v(y(x),t) = v(y,t)$$

Q2,2

$$\left[ \frac{\partial^2 v}{\partial t^2} - \frac{12}{\sqrt{2}} \frac{\partial^2 v}{\partial y^2} = 0 \right.$$

looks like (\*)

$$v(y,0) = f(\sqrt{2}y)$$

$$v_t(y,0) = 0$$

$$v(y,t) = \frac{1}{2} (f(\sqrt{2}(y+t)) + f(\sqrt{2}(y-t)))$$

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$$u(x,t) = v(y(x),t)$$

$$= \frac{1}{2} (f(\sqrt{2}(\frac{1}{\sqrt{2}}x+t)) + f(\sqrt{2}(\frac{1}{\sqrt{2}}x-t)))$$

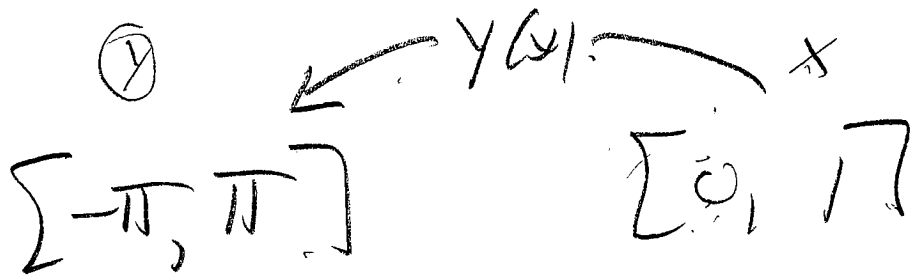
$$= \frac{1}{2} (f(x+\sqrt{2}t) + f(x-\sqrt{2}t))$$

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This solves (\*)

know on  
 $[-\pi, \pi]$

Solve on Q3.1  
 $[0, 1]$



$$y(0) = -\pi$$
$$y(1) = \pi$$

$$y(x) = 2\pi(x-0) - \pi$$

$$\frac{d^2 u}{dx^2} = \lambda u \quad x \in [0, 1]$$

$$u(x) = v(y(x))$$

Chain rule

$$\frac{d}{dx} = \frac{dy}{dx} \frac{d}{dy} = 2\pi \frac{d}{dy}$$

$$\frac{d^2}{dx^2} = (2\pi)^2 \frac{d^2}{dy^2}$$

$y \in [-\pi, \pi]$   
 $v(y) = u(x)$

$$(2\pi)^2 \frac{d^2 v}{dy^2} = \lambda v$$

$$\frac{d^2 v}{dy^2} = \tilde{\lambda} v \quad \tilde{\lambda} = \frac{\lambda}{(2\pi)^2}$$

I can solve.