$$(*) \int_{-\pi}^{\pi} \sin(mx)\sin(nx) = \begin{cases} 0 & n \neq m \\ \pi & n = m \end{cases} \quad (?)$$ $\boxed{Q1.1}$

$m, n \in \mathbb{N} \cup \{0\}$

$$(**) \int_{-\pi}^{\pi} \cos(mx)\cos(nx) = \begin{cases} 0 & n \neq m \\ \pi & n = m > 0 \\ 2\pi & n = m = 0 \end{cases}$$

$$(***) \int_{-\pi}^{\pi} \cos(mx)\sin(nx)\,dx = 0.$$

$\rightarrow$ $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

$$\frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2} = \cancel{2}\sin(\alpha)\sin(\beta) \quad (*)$$

$$(*) = \frac{1}{2}\int_{-\pi}^{\pi} \cancel{2}\big(\cos((m-n)x) - \cos((m+n)x)\big)\,dx$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} \cos((m-n)x) - \frac{1}{2}\int_{-\pi}^{\pi} \cos((m+n)x)$$

$\boxed{m \neq n}$ Q1.2

$$= \frac{1}{2} \frac{\sin((m-n)x)}{m-n} \Bigg|_{-\pi}^{\pi} - \frac{1}{2} \frac{\sin((m+n)x)}{m+n} \Bigg|_{-\pi}^{\pi}$$

$$= \frac{1}{2} \frac{2\sin((m-n)\pi)}{m-n} - \frac{1}{2} \frac{2\sin((m+n)\pi)}{m+n} = 0.$$

$m = n$

$$(*) = \frac{1}{2}\int_{-\pi}^{\pi} 1\, dx - \cancel{\frac{1}{2}\int_{-\pi}^{\pi} \cos(2mx)\, dx} \to 0$$

$$= \pi$$

to $(**)$

$$\frac{\cos(\alpha-\beta) + \cos(\alpha+\beta)}{2} = \cancel{2}\cos(\alpha)\cos(\beta) + 0$$

$$(**) = \frac{1}{2}\int_{-\pi}^{\pi} \cos((m-n)x) + \cos((m+n)x)\ dx$$

$=$ similar to above. $m \neq n > 0$

$** = \frac{1}{2} \cdot 2\pi + 0 \quad m = n \quad \pi$ | $(**) = 2\pi$ $m = n = 0$

$\boxed{Q\ 1.3}$

to do ~~($***$)~~ use

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$$
$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \sin(\beta)\cos(\alpha)$$

$$\frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2} = \cancel{2}\sin(\alpha)\cos(\beta)$$

$$(***) = \frac{1}{2}\int_{-\pi}^{\pi} \sin((n+m)x) + \sin((n-m)x)\,dx$$

$\underline{n \neq m}$

$$= \frac{1}{2}\left[-\frac{\cos((n+m)x)}{n+m}\Big|_{-\pi}^{\pi} - \frac{\cos((n-m)x)}{(n-m)}\Big|_{-\pi}^{\pi}\right]$$

$$= \frac{1}{2}\left[-\left\{\frac{\cos((n+m)\pi) - \cos((n+m)\pi)}{n+m}\right\} - \left\{\frac{\cos((n-m)\pi) - \cos((n-m)\pi)}{n-m}\right\}\right]$$

$$= \frac{1}{2}\left[-\frac{0}{n+m} - \frac{0}{n-m}\right] = 0$$

Q1.4

$n = m$

$$\psi\psi^* = \frac{1}{2}\int_{-\pi}^{\pi} \sin(2nx) \quad + 0$$

$$= \frac{1}{2} \; -\frac{\cos(2nx)}{2n}\Bigg|_{-\pi}^{\pi} = 0$$

Q2.1

Heat Eqn

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Wave Eqn

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$$

Laplace's Eqn

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Q 2,2

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$ 2D Heat Eqn.

$$\frac{\partial u}{\partial t} = \underline{\underline{\nabla^2 u}}$$

$$\boxed{\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}}$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} - \nabla^2 u = 0}$$ 2D Wave Eqn.