Solve

Q1.1

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$u(-\pi,t) = u(\pi,t)$$
$$u_x(-\pi,t) = u_x(\pi,t)$$

2π periodic B.C.

$$u(x,0) = |x| \qquad \text{I.C.}$$

Look for a soln as a linear comb of efuncts $\frac{\partial^2}{\partial x^2}$ with 2π period B.C.

I.e. $$u(x,t) = \alpha_0(t)\cdot 1 + \sum_{n=1}^{\infty} \alpha_n(t)\cos(nx) + \beta_n(t)\sin(nx)$$

This solves the B.C. for free

These are the efuncts.

$$\frac{\partial u}{\partial t} = \frac{\partial \alpha_0}{\partial t}\cdot 1 + \sum_{n=1}^{\infty} \frac{\partial \alpha_n}{\partial t}\cos(nx) + \frac{\partial \beta_n}{\partial t}\sin(nx)$$

|| for all x,t

$$\frac{\partial^2 u}{\partial x^2} = \alpha_0\cdot 0\cdot 1 + \sum_{n=1}^{\infty} -n^2\alpha_n\cos(nx) - n^2\beta_n\sin(nx)$$

$$\frac{\partial \alpha_0}{\partial t} = 0\,\alpha_0, \quad \forall n=1,2,\ldots \quad \frac{\partial \alpha_n}{\partial t} = -n^2\alpha_n, \quad \frac{\partial \beta_n}{\partial t} = -n^2\beta_n$$

Q 1.2

We've turned solving the PDE into solving a "bunch" of ODE,

$$\frac{d\alpha_0}{dt} = 0\,\alpha_0 \Rightarrow \alpha_0(t) = a_0 \text{ (a constant)}$$

$$\frac{d\alpha_n}{dt} = -n^2\alpha_n \Rightarrow \alpha_n(t) = a_n e^{-n^2 t}$$

This is constants

$$\frac{d\beta_n}{dt} = -n^2\beta_n \Rightarrow \beta_n(t) = b_n e^{-n^2 t}$$

$$u(x,t) = a_0 \cdot 1 + \sum_{n=1}^{\infty} e^{-n^2 t}\left(a_n\cos(nx) + b_n\sin(nx)\right)$$

(**)

This satisfies the B.C.s but also the PDE.

Still need to set I.C.

need to determine constants a_0, a_n, b_n

$$|x| = u(x,0) = a_0 + \sum_{n=1}^{\infty} \cancel{e^{-n^2\cdot 0}}^{\,1} \left(a_n\cos(nx) + b_n\sin(nx)\right)$$

Need to determine 2π period F.S. for $|x|$.

Q1,3

From Before

$$|x| \sim \frac{\pi}{2} + \sum_{n=1}^{\infty}\left(\frac{2(1-\cos(n\pi))}{n^2\pi}\cos(nx) + 0\,\sin(nx)\right)$$

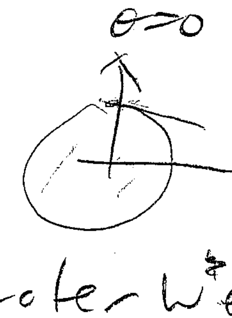
$$a_0 = \pi/2$$

$$a_n = \frac{2(1-\cos(n\pi))}{n^2\pi} \qquad n=1,2,\ldots$$

$$b_n = 0$$

(Plug in to ✱ on prev page)

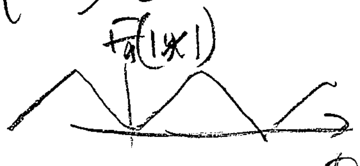
$$\boxed{u(x,t) = \frac{\pi}{2} + \sum_{n=1}^{\infty} e^{-n^2 t}\left(\frac{2(1-\cos(n\pi))}{n^2\pi}\right)\cos(nx)}$$



That's it!

Later we'll solve HE on a disk

as $t \to \infty$ $\quad u(x,t) \to \dfrac{\pi}{2}$



$\boxed{\dfrac{\partial u}{\partial t} = \nabla^2 u}$ This HE on RING!

Welll Do This Later

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Q2.1

This says to

$u(0,t)=0 \quad u(1,t)=0$ use $\sum_{n=1}^{\infty} a_n(t)\sin(n\pi x)$

$u(x,0)=|x|$

$$\boxed{\begin{aligned} &\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \\ &u(0,t)=280^\circ K \quad u(1,t)=300^\circ K \end{aligned}}$$

$u(x,t) = \underline{S(x)} + v(x,t)$

$$0 = \frac{d^2 S}{dx^2}$$

2 pt B.V.P.

$S(0)=280, \quad S(1)=300$

$S(x) = a + bx$

$$\boxed{S(x) = 280 + 20 \cdot x}$$

We'll do This later

Q2.2

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial (S+V)}{\partial t} = \frac{\partial^2 (S+V)}{\partial x^2}$$

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}$$

What kind of B.C.s does V need to satisfy?

$u(x,t) \to S(x)$ as $t \to \infty$

$$280 = u(0,t) = S(0) + V(0,t)$$

$$0 = V(0,t)$$

$$300 = u(1,t) = S(1) + V(1,t)$$

$$0 = V(1,t)$$

$$V(x,0) = f(x) - S(x)$$

$$f(x) = u(x,0) = S(x) + V(x,0)$$

You are to know (Q3.)

$$f(x) = \sum_{n=1}^{\infty} a_n \sin(n\pi x)$$ I

$$f(x) = \sum_{n=0}^{\infty} a_n \cos(n\pi x)$$ II

$$f(x) = \sum_{n=1}^{\infty} a_n \sin((n-1/2)\pi x)$$ III

$$f(x) = \sum_{n=1}^{\infty} a_n \cos((n-1/2)\pi x)$$ IV

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$ V

Called the Fourier Coeffs.