(Q1.)

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = f(x)$$

$$u_t(x,0) = g(x)$$

Wave eqn for general c by change of variable

$$y = \frac{x}{c} \qquad u(x,t) = W(y,t)$$

$$\frac{\partial^2 W}{\partial t^2} = \frac{\partial^2 W}{\partial y^2}$$

$$W(y,0) = u(cy,0) = f(cy) = \tilde{f}(y)$$

$$W_t(y,0) = u_t(cy,0) = g(cy) = \tilde{g}(y)$$

From class

$$W(y,t) = \frac{1}{2}\left(\tilde{f}(y+t) + \tilde{f}(y-t)\right) + \frac{1}{2}\int_{y-t}^{y+t} \tilde{g}(s)\,ds$$

Q1.2

$$u(x,t) = w\left(\frac{x}{c}, t\right)$$

$$= \frac{1}{2}\left(\tilde{f}\left(\frac{x}{c}+t\right) + \tilde{f}\left(\frac{x}{c}-t\right)\right) + \frac{1}{2}\int_{\frac{x}{c}-t}^{\frac{x}{c}+t} \tilde{g}(s)\,ds$$

$$= \frac{1}{2}\left(f\left(c\left(\frac{x}{c}+t\right)\right) + f\left(c\left(\frac{x}{c}-t\right)\right)\right)$$

$z = cs = c\left(\frac{x}{c}-t\right)$

$$+ \frac{1}{2}\int_{\frac{x}{c}-t}^{\frac{x}{c}+t} g(\underline{cs})\,ds$$

$cs = z$

$ds = \frac{1}{c}\,dz$

$$= \frac{1}{2}\left(f(x+ct) + f(x-ct)\right) + \frac{1}{2c}\int_{x-ct}^{x+ct} g(z)\,dz$$