$\boxed{Q1.1}$

Parsev $\left(f(x)\right)^2 = \left(\sum_{n=1}^{\infty} \underline{\underline{a_n}} \sin(n\pi x)\right)^2$

$$= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} a_n \sin(n\pi x)\, a_m \sin(m\pi x)$$

$$\int_0^1 (f(x))^2 dx = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} a_n a_m \underbrace{\int_0^1 \sin(n\pi x) \sin(m\pi x)\, dx}_{\begin{cases} 0 & n \neq m \\ 1/2 & n = m \end{cases}}$$

$$= \sum_{n=1}^{\infty} (a_n)^2 \cdot \frac{1}{2}$$

$$\boxed{\int_0^1 (f(x))^2 dx = \frac{1}{2} \sum_{n=1}^{\infty} a_n^2 \quad \text{(P)}}$$

1a $x \sim \sum_{n=1}^{\infty} a_n \sin(n\pi x)$

$$a_n = 2\int_0^1 y \sin(n\pi y)\,dy$$

u $\quad dv$

$$= 2\left[-y\frac{\cos(n\pi y)}{n\pi}\Big|_0^1 + \cancel{\int_0^1 \frac{\cos(n\pi y)}{n\pi}\,dy} \right]$$

$$= \frac{2(-\cos(n\pi))}{n\pi} = \frac{2(-1)^{n+1}}{n\pi}$$

$$x \sim \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n\pi}\sin(n\pi x)$$

Use part a to determine value for

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \left(\pi^2/6\right)$$

Q1.3

From Parseval

$$\int_0^1 x^2\,dx = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{2(-1)^{n+1}}{n\pi}\right)^2$$

$$\frac{1}{3} = \frac{1}{2}\,\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty}\frac{1}{n^2}$$