MATH 3363 EXAM I. Sanders Spring 2008

This exam has 5 problems, and all 5 problems will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless otherwise indicated.

1. Determine whether or not the following are linear operators.

(a) $\mathcal{L}(u) = \dfrac{du}{dx} + u.$ (c) $\mathcal{L}(u) = u\dfrac{du}{dx}.$

(b) $\mathcal{L}(u) = \dfrac{du}{dx} + 1.$ (d) $\mathcal{L}(u) = x^2\dfrac{d^2u}{dx^2} + e^x u.$

2. Determine all eigenvalues and eigenfunctions to the following.

$$\frac{d^2u}{dx^2} = \lambda u, \quad u_x(0) = 0 \;\; u_x(1) = 0.$$

You may assume all eigenvalues are real but nothing else. These must be correctly enumerated to receive full credit.

3. The Fourier cosine series for a function f has the form $f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x)$.

(a) Show $\{\cos(n\pi x)\}_{n=0}^{\infty}$ forms an orthogonal set with respect to $(f, g) = \int_0^1 f(x)g(x)\,dx$.

(b) Derive the formulae for the Fourier coefficients a_0 and a_n, $n = 1, 2\ldots$, in terms of integrals involving f.

(c) Compute the cosine series for $f(x) = \cos^2(\pi x)$. (Hint: $\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$.)

(d) Compute the cosine series for $f(x) = x$. (Answer: $x \sim \frac{1}{2} + 2\sum_{n=1}^{\infty} \frac{((-1)^n - 1)}{(n\pi)^2}\cos(n\pi x)$.)

4. Solve the *heat equation* $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ on $0 < x < 1$, $t > 0$, with boundary conditions $u_x(0, t) = 0$, $u_x(1, t) = 0$ and initial condition $u(x, 0) = f(x)$, when

(a) $f(x) = \cos(2\pi x) + 3\cos(3\pi x)$ (b) $f(x) = x$

5. Solve the *wave equation* $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$ on $0 < x < 1$, $t > 0$, with boundary conditions $u_x(0, t) = 0$, $u_x(1, t) = 0$ and initial conditions $u(x, 0) = f(x)$, $u_t(x, 0) = 0$, when

(a) $f(x) = \cos(2\pi x) + 3\cos(3\pi x)$ (b) $f(x) = x$