

# TEST FRAMEWORK FOR FIELD 135: MATHEMATICS 8–12

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**Domain I**    **Number Concepts**  
(approximately 14% of the test)

**Standards Assessed:**

**Mathematics Standard I:**

Number Concepts: The mathematics teacher understands and uses numbers, number systems and their structure, operations and algorithms, quantitative reasoning, and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

**Domain II**    **Patterns and Algebra**  
(approximately 33% of the test)

**Standards Assessed:**

**Mathematics Standard II:**

Patterns and Algebra: The mathematics teacher understands and uses patterns, relations, functions, algebraic reasoning, analysis, and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

**Domain III**    **Geometry and Measurement**  
(approximately 19% of the test)

**Standards Assessed:**

**Mathematics Standard III:**

Geometry and Measurement: The mathematics teacher understands and uses geometry, spatial reasoning, measurement concepts and principles, and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

**Domain IV**    **Probability and Statistics**  
(approximately 14% of the test)

**Standards Assessed:**

**Mathematics Standard IV:**

Probability and Statistics: The mathematics teacher understands and uses probability and statistics, their applications, and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

**Domain V Mathematical Processes and Perspectives  
(approximately 10% of the test)**

**Standards Assessed:**

**Mathematics Standards V and VI:**

**Mathematical Processes:** The mathematics teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics, and to communicate mathematically.

**Mathematical Perspectives:** The mathematics teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics, and the evolving nature of mathematics and mathematical knowledge.

**Domain VI Mathematical Learning, Instruction, and Assessment  
(approximately 10% of the test)**

**Standards Assessed:**

**Mathematics Standards VII and VIII:**

**Mathematical Learning and Instruction:** The mathematics teacher understands how children learn and develop mathematical skills, procedures, and concepts, knows typical errors students make, and uses this knowledge to plan, organize, and implement instruction; to meet curriculum goals; and to teach all students to understand and use mathematics.

**Mathematical Assessment:** The mathematics teacher understands assessment and uses a variety of formal and informal assessment techniques appropriate to the learner on an ongoing basis to monitor and guide instruction and to evaluate and report student progress.

## DOMAIN I—NUMBER CONCEPTS

### Competency 001

**The teacher understands the real number system and its structure, operations, algorithms, and representations.**

The beginning teacher:

- Understands the concepts of place value, number base, and decimal representations of real numbers.
- Understands the algebraic structure and properties of the real number system and its subsets (e.g., real numbers as a field, integers as an additive group).
- Describes and analyzes properties of subsets of the real numbers (e.g., closure, identities).
- Selects and uses appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- Uses a variety of models (e.g., geometric, symbolic) to represent operations, algorithms, and real numbers.
- Uses real numbers to model and solve a variety of problems.
- Uses deductive reasoning to simplify and justify algebraic processes.
- Demonstrates how some problems that have no solution in the integer or rational number systems have solutions in the real number system.

### Competency 002

**The teacher understands the complex number system and its structure, operations, algorithms, and representations.**

The beginning teacher:

- Demonstrates how some problems that have no solution in the real number system have solutions in the complex number system.
- Understands the properties of complex numbers (e.g., complex conjugate, magnitude/modulus, multiplicative inverse).
- Understands the algebraic structure of the complex number system and its subsets (e.g., complex numbers as a field, complex addition as vector addition).
- Selects and uses appropriate representations of complex numbers (e.g., vector, ordered pair, polar, exponential) for particular situations.
- Describes complex number operations (e.g., addition, multiplication, roots) using symbolic and geometric representations.

**Competency 003**

**The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.**

The beginning teacher:

- Applies ideas from number theory (e.g., prime numbers and factorization, the Euclidean algorithm, divisibility, congruence classes, modular arithmetic, the fundamental theorem of arithmetic) to solve problems.
- Applies number theory concepts and principles to justify and prove number relationships.
- Compares and contrasts properties of vectors and matrices with properties of number systems (e.g., existence of inverses, non-commutative operations).
- Uses properties of numbers (e.g., fractions, decimals, percents, ratios, proportions) to model and solve real-world problems.
- Applies counting techniques such as permutations and combinations to quantify situations and solve problems.
- Uses estimation techniques to solve problems and judges the reasonableness of solutions.

**DOMAIN II—PATTERNS AND ALGEBRA****Competency 004**

**The teacher uses patterns to model and solve problems and formulate conjectures.**

The beginning teacher:

- Recognizes and extends patterns and relationships in data presented in tables, sequences, or graphs.
- Uses methods of recursion and iteration to model and solve problems.
- Uses the principle of mathematical induction.
- Analyzes the properties of sequences and series (e.g., Fibonacci, arithmetic, geometric) and uses them to solve problems involving finite and infinite processes.
- Understands how sequences and series are applied to solve problems in the mathematics of finance (e.g., simple, compound, and continuous interest rates; annuities).

**Competency 005****The teacher understands attributes of functions, relations, and their graphs.**

The beginning teacher:

- Understands when a relation is a function.
- Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.
- Understands that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).
- Identifies and analyzes even and odd functions, one-to-one functions, inverse functions, and their graphs.
- Applies basic transformations [e.g.,  $k f(x)$ ,  $f(x) + k$ ,  $f(x - k)$ ,  $f(kx)$ ,  $|f(x)|$ ] to a parent function,  $f$ , and describes the effects on the graph of  $y = f(x)$ .
- Performs operations (e.g., sum, difference, composition) on functions, finds inverse relations, and describes results symbolically and graphically.
- Uses graphs of functions to formulate conjectures of identities [e.g.,  $y = x^2 - 1$  and  $y = (x - 1)(x + 1)$ ,  $y = \log x^3$  and  $y = 3 \log x$ ,  $y = \sin(x + \frac{\pi}{2})$  and  $y = \cos x$ ].

**Competency 006****The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.**

The beginning teacher:

- Understands the concept of slope as a rate of change and interprets the meaning of slope and intercept in a variety of situations.
- Writes equations of lines given various characteristics (e.g., two points, a point and slope, slope and  $y$ -intercept).
- Applies techniques of linear and matrix algebra to represent and solve problems involving linear systems.
- Analyzes the zeros (real and complex) of quadratic functions.
- Makes connections between the  $y = ax^2 + bx + c$  and the  $y = a(x - h)^2 + k$  representations of a quadratic function and its graph.
- Solves problems involving quadratic functions using a variety of methods (e.g., factoring, completing the square, using the quadratic formula, using a graphing calculator).
- Models and solves problems involving linear and quadratic equations and inequalities using a variety of methods, including technology.

**Competency 007**

**The teacher understands polynomial, rational, radical, absolute value, and piecewise functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.**

The beginning teacher:

- Recognizes and translates among various representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value, and piecewise functions.
- Describes restrictions on the domains and ranges of polynomial, rational, radical, absolute value, and piecewise functions.
- Makes and uses connections among the significant points (e.g., zeros, local extrema, points where a function is not continuous or not differentiable) of a function, the graph of the function, and the function's symbolic representation.
- Analyzes functions in terms of vertical, horizontal, and slant asymptotes.
- Analyzes and applies the relationship between inverse variation and rational functions.
- Solves equations and inequalities involving polynomial, rational, radical, absolute value, and piecewise functions using a variety of methods (e.g., tables, algebraic methods, graphs, use of a graphing calculator), and evaluates the reasonableness of solutions.
- Models situations using polynomial, rational, radical, absolute value, and piecewise functions and solves problems using a variety of methods, including technology.

### Competency 008

**The teacher understands exponential and logarithmic functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.**

The beginning teacher:

- Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of exponential and logarithmic functions.
- Recognizes and uses connections among significant characteristics (e.g., intercepts, asymptotes) of a function involving exponential or logarithmic expressions, the graph of the function, and the function's symbolic representation.
- Understands the relationship between exponential and logarithmic functions and uses the laws and properties of exponents and logarithms to simplify expressions and solve problems.
- Uses a variety of representations and techniques (e.g., numerical methods, tables, graphs, analytic techniques, graphing calculators) to solve equations, inequalities, and systems involving exponential and logarithmic functions.
- Models and solves problems involving exponential growth and decay.
- Uses logarithmic scales (e.g., Richter, decibel) to describe phenomena and solve problems.
- Uses exponential and logarithmic functions to model and solve problems involving the mathematics of finance (e.g., compound interest).
- Uses the exponential function to model situations and solve problems in which the rate of change of a quantity is proportional to the current amount of the quantity [i.e.,  $f'(x) = k f(x)$ ].

### Competency 009

**The teacher understands trigonometric and circular functions, analyzes their algebraic and graphical properties, and uses them to model and solve problems.**

The beginning teacher:

- Analyzes the relationships among the unit circle in the coordinate plane, circular functions, and the trigonometric functions.
- Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of trigonometric functions and their inverses.
- Recognizes and uses connections among significant properties (e.g., zeros, axes of symmetry, local extrema) and characteristics (e.g., amplitude, frequency, phase shift) of a trigonometric function, the graph of the function, and the function's symbolic representation.
- Understands the relationships between trigonometric functions and their inverses and uses these relationships to solve problems.
- Uses trigonometric identities to simplify expressions and solve equations.
- Models and solves a variety of problems (e.g., analyzing periodic phenomena) using trigonometric functions.
- Uses graphing calculators to analyze and solve problems involving trigonometric functions.

**Competency 010****The teacher understands and solves problems using differential and integral calculus.**

The beginning teacher:

- Understands the concept of limit and the relationship between limits and continuity.
- Relates the concept of average rate of change to the slope of the secant line and the concept of instantaneous rate of change to the slope of the tangent line.
- Uses the first and second derivatives to analyze the graph of a function (e.g., local extrema, concavity, points of inflection).
- Understands and applies the fundamental theorem of calculus and the relationship between differentiation and integration.
- Models and solves a variety of problems (e.g., velocity, acceleration, optimization, related rates, work, center of mass) using differential and integral calculus.
- Analyzes how technology can be used to solve problems and illustrate concepts involving differential and integral calculus.

**DOMAIN III—GEOMETRY AND MEASUREMENT****Competency 011****The teacher understands measurement as a process.**

The beginning teacher:

- Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.
- Applies formulas for perimeter, area, surface area, and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.
- Recognizes the effects on length, area, or volume when the linear dimensions of plane figures or solids are changed.
- Applies the Pythagorean theorem, proportional reasoning, and right triangle trigonometry to solve measurement problems.
- Relates the concept of area under a curve to the limit of a Riemann sum.
- Uses integral calculus to compute various measurements associated with curves and regions (e.g., area, arc length) in the plane and measurements associated with curves, surfaces, and regions in three-space.

**Competency 012**

**The teacher understands geometries, in particular Euclidean geometry, as axiomatic systems.**

The beginning teacher:

- Understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).
- Uses properties of points, lines, planes, angles, lengths, and distances to solve problems.
- Applies the properties of parallel and perpendicular lines to solve problems.
- Uses properties of congruence and similarity to explore geometric relationships, justify conjectures, and prove theorems.
- Describes and justifies geometric constructions made using compass and straightedge, reflection devices, and other appropriate technologies.
- Demonstrates an understanding of the use of appropriate software to explore attributes of geometric figures and to make and evaluate conjectures about geometric relationships.
- Compares and contrasts the axioms of Euclidean geometry with those of non-Euclidean geometry (i.e., hyperbolic and elliptic geometry).

**Competency 013**

**The teacher understands the results, uses, and applications of Euclidean geometry.**

The beginning teacher:

- Analyzes the properties of polygons and their components.
- Analyzes the properties of circles and the lines that intersect them.
- Uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).
- Computes the perimeter, area, and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc length, area of sectors).
- Analyzes cross-sections and nets of three-dimensional shapes.
- Uses top, front, side, and corner views of three-dimensional shapes to create complete representations and solve problems.
- Applies properties of two- and three-dimensional shapes to solve problems across the curriculum and in everyday life.

#### **Competency 014**

**The teacher understands coordinate, transformational, and vector geometry and their connections.**

The beginning teacher:

- Identifies transformations (i.e., reflections, translations, glide-reflections, rotations, dilations) and explores their properties.
- Uses the properties of transformations and their compositions to solve problems.
- Uses transformations to explore and describe reflectional, rotational, and translational symmetry.
- Applies transformations in the coordinate plane.
- Applies concepts and properties of slope, midpoint, parallelism, perpendicularity, and distance to explore properties of geometric figures and solve problems in the coordinate plane.
- Uses coordinate geometry to derive and explore the equations, properties, and applications of conic sections (i.e., lines, circles, hyperbolas, ellipses, parabolas).
- Relates geometry and algebra by representing transformations as matrices and uses this relationship to solve problems.
- Explores the relationship between geometric and algebraic representations of vectors and uses this relationship to solve problems.

### **DOMAIN IV—PROBABILITY AND STATISTICS**

#### **Competency 015**

**The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns, and describe departures from patterns.**

The beginning teacher:

- Selects and uses an appropriate measurement scale (i.e., nominal, ordinal, interval, ratio) to answer research questions and analyze data.
- Organizes, displays, and interprets data in a variety of formats (e.g., tables, frequency distributions, scatter plots, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).
- Applies concepts of center, spread, shape, and skewness to describe a data distribution.
- Understands measures of central tendency (i.e., mean, median, mode) and dispersion (i.e., range, interquartile range, variance, standard deviation).
- Applies linear transformations (i.e., translating, stretching, shrinking) to convert data and describes the effect of linear transformations on measures of central tendency and dispersion.
- Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers, and measures of central tendency and dispersion.
- Supports arguments, makes predictions, and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.

**Competency 016****The teacher understands concepts and applications of probability.**

The beginning teacher:

- Understands how to explore concepts of probability through sampling, experiments, and simulations, and generates and uses probability models to represent situations.
- Uses the concepts and principles of probability to describe the outcomes of simple and compound events.
- Determines probabilities by constructing sample spaces to model situations.
- Solves a variety of probability problems using combinations and permutations.
- Solves a variety of probability problems using ratios of areas of geometric regions.
- Calculates probabilities using the axioms of probability and related theorems and concepts such as the addition rule, multiplication rule, conditional probability, and independence.
- Understands expected value, variance, and standard deviation of probability distributions (e.g., binomial, geometric, uniform, normal).
- Applies concepts and properties of discrete and continuous random variables to model and solve a variety of problems involving probability and probability distributions (e.g., binomial, geometric, uniform, normal).

**Competency 017****The teacher understands the relationships among probability theory, sampling, and statistical inference, and how statistical inference is used in making and evaluating predictions.**

The beginning teacher:

- Applies knowledge of designing, conducting, analyzing, and interpreting statistical experiments to investigate real-world problems.
- Analyzes and interprets statistical information (e.g., the results of polls and surveys) and recognizes misleading as well as valid uses of statistics.
- Understands random samples and sample statistics (e.g., the relationship between sample size and confidence intervals, biased or unbiased estimators).
- Makes inferences about a population using binomial, normal, and geometric distributions.
- Describes and analyzes bivariate data using various techniques (e.g., scatterplots, regression lines, outliers, residual analysis, correlation coefficients).
- Understands how to transform nonlinear data into linear form in order to apply linear regression techniques to develop exponential, logarithmic, and power regression models.
- Uses the law of large numbers and the central limit theorem in the process of statistical inference.
- Estimates parameters (e.g., population mean and variance) using point estimators (e.g., sample mean and variance).
- Understands principles of hypotheses testing.

## DOMAIN V—MATHEMATICAL PROCESSES AND PERSPECTIVES

### Competency 018

**The teacher understands mathematical reasoning and problem solving.**

The beginning teacher:

- Understands the nature of proof, including indirect proof, in mathematics.
- Applies correct mathematical reasoning to derive valid conclusions from a set of premises.
- Uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.
- Uses formal and informal reasoning to justify mathematical ideas.
- Understands the problem-solving process (i.e., recognizing that a mathematical problem can be solved in a variety of ways, selecting an appropriate strategy, evaluating the reasonableness of a solution).
- Evaluates how well a mathematical model represents a real-world situation.

### Competency 019

**The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.**

The beginning teacher:

- Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of a circle as a quadratic function of the radius, probability as the ratio of two areas, area of a plane region as a definite integral).
- Understands how mathematics is used to model and solve problems in other disciplines (e.g., art, music, science, social science, business).
- Translates mathematical ideas between verbal and symbolic forms.
- Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).
- Understands the use of visual media, such as graphs, tables, diagrams, and animations, to communicate mathematical information.
- Uses appropriate mathematical terminology to express mathematical ideas.

## DOMAIN VI —MATHEMATICAL LEARNING, INSTRUCTION, AND ASSESSMENT

### Competency 020

**The teacher understands how children learn mathematics and plans, organizes, and implements instruction using knowledge of students, subject matter, and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

The beginning teacher:

- Applies research-based theories of learning mathematics to plan appropriate instructional activities for all students.
- Understands how students differ in their approaches to learning mathematics.
- Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.
- Understands how learning may be enhanced through the use of manipulatives, technology, and other tools (e.g., stop watches, scales, rulers).
- Understands how to provide instruction along a continuum from concrete to abstract.
- Understands a variety of instructional strategies and tasks that promote students' abilities to do the mathematics described in the TEKS.
- Understands how to create a learning environment that provides all students, including English Language Learners, with opportunities to develop and improve mathematical skills and procedures.
- Understands a variety of questioning strategies to encourage mathematical discourse and to help students analyze and evaluate their mathematical thinking.
- Understands how to relate mathematics to students' lives and a variety of careers and professions.

### Competency 021

**The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.**

The beginning teacher:

- Understands the purpose, characteristics, and uses of various assessments in mathematics, including formative and summative assessments.
- Understands how to select and develop assessments that are consistent with what is taught and how it is taught.
- Understands how to develop a variety of assessments and scoring procedures consisting of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns.
- Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor, and modify instruction to improve mathematical learning for all students, including English Language Learners.