Name: 

Section(instructor): 

Score: 

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Instruction:

• There are 16 numbered problems. You must solve the problems 1-10, and you need to solve four problems in 11-16. Each problem is 10 points, and the maximum total is 140.

• Please circle the index numbers of four problems in 11-16 which you want to be graded.

• Show all work clearly to receive full credit.

• Use the back of a page if you need more space, but clearly indicate in the space following the problem where extra work can be found.

• Be sure to neatly cross out all work that you do not want to be considered. Otherwise it may have the effect of reducing your score.

Formulas:

\[
\sin(2x) = 2 \sin x \cos x \quad \cos(2x) = \cos^2 x - \sin^2 x
\]

\[
\sin^2 x = \frac{1 - \cos(2x)}{2} \quad \cos^2 x = \frac{1 + \cos(2x)}{2}
\]

\[
\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C \quad \int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C
\]
1. (a) Find the area enclosed by the given curves: \( y = 3x + 4 \) and \( y = 2 - x^2 \).

(b) Consider the function \( y = \sqrt{e^{2x} - 1} \). Set up an integral that gives the arc length of the curve from \((0, 0)\) to \((1, \sqrt{e^2 - 1})\). (But do not evaluate the integral.)

2. Let \( R \) be the region bounded by \( y = x^2 \) and \( x = 2y \) in the first quadrant, and let \( S \) be the solid obtained by rotating \( R \) about the \( x \)-axis.

(a) Set up, but do not evaluate, the integral(s) required to compute the volume of \( S \) by the washer(disk) method with \( x \) as the variable of integral.

(b) Set up, but do not evaluate, the integral(s) required to compute the volume of \( S \) by the cylindrical shells method with \( y \) as the variable of integral.
3. Evaluate the following integrals. You must show all necessary steps including substitutions, integral by parts, partial fractions, etc.

   (a) \[ \int \frac{x^3}{\sqrt{9-x^2}} \, dx; \]

   (b) \[ \int x^6 \ln x \, dx. \]

4. Evaluate the integral \[ \int \frac{1}{(x-1)(x^2+9)} \, dx. \]
5. Determine whether each improper integral is convergent or divergent. Evaluate the integral if it is convergent.

(a) \( \int_0^{\infty} \frac{1}{2x + 5} \, dx \).

(b) \( \int_0^{3} \frac{1}{\sqrt{x}} \, dx \).

6. Find the solutions of the following differential equations:

(a) \( \frac{dy}{dx} = \sin(x)y^2 \).

(b) \( \frac{dy}{dx} + \frac{y}{x} = 3x^2 - 2x, \ y(1) = 1 \).
7. Determine whether the series is convergent or divergent. If it is convergent, find its sum:

(a) \[ \sum_{n=1}^{\infty} \frac{3^{n+1} + 2^n}{5^n} \]

(b) \[ \sum_{n=1}^{\infty} \ln \frac{n}{n+1} \]

8. Determine whether the following positive series are convergent or divergent.

(a) \[ \sum_{n=1}^{\infty} \frac{n^3}{3^n} ; \]

(b) \[ \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+4} ; \]
9. (a) Determine whether the following series is absolutely convergent, conditionally convergent
or divergent: \[ \sum_{n=1}^{\infty} (-1)^{n-1}\frac{\ln n}{n}. \]

(b) Find the interval of convergence of the power series \[ \sum_{n=1}^{\infty} (-1)^n \frac{(x + 2)^n}{n2^n}. \]

10. (a) Find the first four terms of the Taylor series (third degree Taylor polynomial) \( T_3(x) \) for \( f(x) = \sqrt{x + 2} \) at \( x = 2 \).

(b) Given that \( \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \) for \(-1 < x < 1\). Find the Maclaurin series of \( \ln(1 - 2x) \)
by integrating an appropriate power series, and find the interval of convergence of the series.
Attention: The following six problems are different applications of calculus. Please solve only four (4) problems from these six (6). If you solve more than four, please mark (circle the index number) which four you would like to be graded.

11. A cylindrical tank of radius 3 ft and height 20 ft is standing on its base on horizontal ground. Gasoline weighing 40 lb/ft$^3$ fills the tank. Find the work required to pump the gas over the top of the tank. (The gravity constant is 32 ft/s$^2$.)

12. Find the centroid of the region bounded by the curves $y = \sin 4x$, $y = 0$, $x = 0$ and $x = \pi/4$. 
13. A trough is filled with a liquid of density 840 kg/m³. The ends of the trough are equilateral triangles with sides 8 meters long and vertex at the bottom. Find the hydrostatic force on one end of the trough.

14. Carbon-14 has a half-life of 5700 years. You are an archaeologist at a site of prehistoric human habitation in North America. A burned fossil stick of maple contains 1 microgram of Carbon-14. A similar piece of maple of the same size burned today contains 8 micrograms of Carbon-14. How many years ago was the fossil stick burned?
15. A tank contains 30kg of salt dissolved in 1000L of water. Brine that contains 0.1kg of salt per liter of water enters the tank at a rate of 10L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. Let $A(t)$ be the amount of salt at time $t$.

(a) Set up a differential equation for $A(t)$, and specify the initial condition $A(0)$.
(b) Solve the equation to find an expression for $A(t)$.

16. According to Newton’s Law of Cooling, the temperature $F(t)$ of a body in a surrounding medium changes at a rate that is proportional to the difference between the temperature of the body and the temperature of the surroundings. It follows that $F(t)$ satisfies a differential equation:

$$\frac{dF}{dt} = k(F - F_s), \quad F(0) = F_0,$$

where $t$ is expressed in minutes, $F_s$ is the temperature in Celsius of the surrounding medium, $F_0$ is the initial temperature of the body, and $k$ is a constant. A hard-boiled egg at $98^\circ C$ is put in a pan under running $10^\circ C$ water to cool. After 5 minutes, the egg’s temperature is found to be $38^\circ C$. How much longer will it take the egg to reach $20^\circ C$?