Exam 2
Math 214
In-Class Portion
SOLUTIONS

1	
2	
3	
4	
5	
6	
Total	

Name: ________________________________

Problem 1.

(a) (2 points) If A is a set, give the definition of a **relation** on A.

A relation R on A is a subset $R \subseteq A \times A$.

(b) (3 points) Define what it means for a relation on A to be an **equivalence relation**. (State the meaning of any properties that you list in your definition)

A relation R on A is an equivalence relation if it has the following three properties:

REFLEXIVE: xRx for all $x \in A$,

SYMMETRIC: xRy implies yRx,

TRANSITIVE: If xRy and yRz, then xRz.

(c) (3 points) Define what it means for a relation on A to be an **partial ordering**. (State the meaning of any properties that you list in your definition)

A relation R on A is a partial ordering if it has the following three properties:

REFLEXIVE: xRx for all $x \in A$,

ANTISYMMETRIC: xRy and yRx implies $x = y$,

TRANSITIVE: If xRy and yRz, then xRz.

Problem 2. Consider the sets

$$A = \{1, 2, 3, 4\} \qquad B = \{a, b, c\} \qquad C = \{\Box, \star\}.$$

Let R be the relation from A to B defined by $R = \{(1, a), (1, c), (2, c), (3, b)\}$.
Let S be the relation from B to A defined by $S = \{(a, 2), (b, 1), (c, 1)$.
Let T be the relation from B to C define by $T = \{(a, \Box), (b, \star)\}$.

(a) (2 points) Compute R^{-1}.

$R^{-1} = \{(a, 1), (c, 1), (c, 2), (b, 3)\}$

(b) (2 points) Compute $S \circ R$.

$S \circ R = \{(1, 2), (1, 1), (2, 1), (3, 1)$.

(c) (2 points) Compute $R \circ S$.

$R \circ S = \{(a, c), (b, a), (b, c), (c, a), (c, c)\}$.

(d) (2 points) Compute $T \circ R$.

$T \circ R = \{(1, \Box), (3, \star)\}$

(e) (2 points) Is $R \circ T$ defined? Why or why not?

No. Because T goes from B to C, while R goes from A to B.

(f) (2 points) Is the relation R the graph of a function? Why or why not?

No. For the value $1 \in A$, there are two values $a, c \in B$ for which $(1, a) \in R$ and $(1, c) \in R$.

(g) (2 points) Is the relation S the graph of a function? Why or why not?

Yes. For each $x \in B$ there exists a unique $y \in A$ such that $(x, y) \in S$.

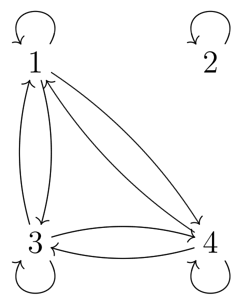
(h) (2 points) Is the relation T the graph of a function? Why or why not?

No. For the value $c \in B$, there is no value $y \in C$ such that $(c, y) \in T$.

Problem 3. Consider the set $A = \{1, 2, 3, 4\}$.

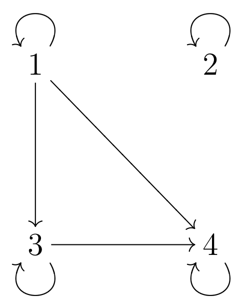
For each of the following digraphs, indicate whether the digraph represents a relation on A that is an **Equivalence Relation**, a **Partial Ordering**, **Both**, or **Neither**.

(a) (2 points)



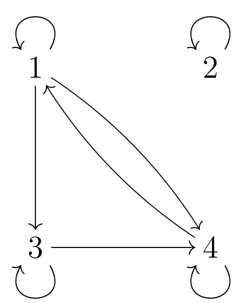
EQUIVALENCE RELATION

(b) (2 points)



PARTIAL ORDERING

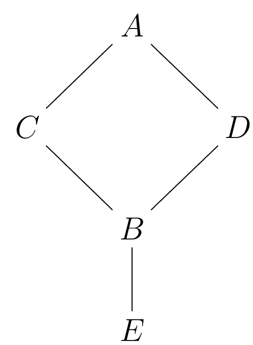
(c) (2 points)



NEITHER

Problem 4. (4 points) Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 3\}$, $C = \{1, 2, 3\}$, $D = \{2, 3, 4\}$, and $E = \{2\}$. Let the collection $\mathcal{A} = \{A, B, C, D, E\}$ be ordered by set inclusion.

Draw a Hasse diagram for $\mathcal{A}$.



Problem 5. Recall that every nonempty subset of $\mathbb{N}$ has a smallest element. If $A \subseteq \mathbb{N}$, we denote the smallest element of A by $\inf(A)$. Let $f : \mathbb{N} \to \mathcal{P}(\mathbb{N})$ by $f(n) = \{n, 2n, 3n, 4n, \ldots\}$. Also let $g : \mathcal{P}(\mathbb{N}) \to \mathbb{N}$ by

$$g(A) = \begin{cases} 1 & \text{if } A = \emptyset \\ \inf(A) & \text{if } A \neq \emptyset. \end{cases}$$

(a) (2 points) Is f injective?

Yes.

(b) (2 points) Is f surjective?

No.

(c) (2 points) Is g injective?

No.

(d) (2 points) Is g surjective?

Yes.

(e) (2 points) For $n \in \mathbb{N}$, find $(g \circ f)(n)$.

$(g \circ f)(n) = n$

(f) (2 points) For $A \in \mathcal{P}(\mathbb{N})$, find $(f \circ g)(A)$.

$$(f \circ g)(A) = \begin{cases} \{\inf(A), 2\inf(A), 3\inf(A), \ldots\} & \text{if } A \neq \emptyset \\ \{1, 2, 3, \ldots\} & \text{if } A = \emptyset \end{cases}$$

Problem 6. Define $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ by $f(x, y) = (x + y, xy)$.

(a) (2 points) Is f injective? Why or why not?

No. We see that $f(2, 3) = f(3, 2)$.

(b) (2 points) Is f surjective? Why or why not?

No. If f was surjective there would exist $(x, y) \in \mathbb{R} \times \mathbb{R}$ such that $f(x, y) = (0, 1)$. But then $x + y = 0$ and $xy = 1$. Solving the first equation for y gives $y = -x$, and plugging this into the second equation gives $-x^2 = 1$, which is a contradiction if $x \in \mathbb{R}$.