

Final Exam  
Math 214  
In-Class Portion  
SOLUTIONS

<b>1</b>	
<b>2</b>	
<b>3</b>	
<b>4</b>	
<b>5</b>	
<b>6</b>	
<b>Total</b>	

Name: \_\_\_\_\_

**Problem 1.** (12 points) Let  $f : A \rightarrow B$  be a function. Identify each of the following statements as **EQUIVALENT** or **NOT EQUIVALENT** to the statement “ $f$  is surjective”.

- (a)  $\text{Ran}(f) = B$ .  
EQUIVALENT
- (b) If  $a \in A$ , then  $b = f(a)$  and  $b \in B$ .  
NOT EQUIVALENT
- (c) For every  $b \in B$  there exists  $a \in A$  such that  $f(a) = b$ .  
EQUIVALENT
- (d)  $B \subseteq \text{Ran}(f)$   
EQUIVALENT
- (e) There exists  $b \in B$  such that  $f(a) = b$  for some  $a \in A$ .  
NOT EQUIVALENT
- (f) The function  $f$  has a left inverse; i.e., there is a function  $g : B \rightarrow A$  such that  $g \circ f = I_A$ .  
NOT EQUIVALENT

**Problem 2.** (12 points) Let  $f : A \rightarrow B$  be a function. Identify each of the following statements as **EQUIVALENT** or **NOT EQUIVALENT** to the statement “ $f$  is injective”.

- (a) If  $x, y \in A$  and  $x = y$ , then  $f(x) = f(y)$ .  
NOT EQUIVALENT
- (b) If  $x, y \in A$  and  $x \neq y$ , then  $f(x) \neq f(y)$ .  
EQUIVALENT
- (c)  $f(x) = f(y)$  and  $x = y$  for all  $x, y \in A$ .  
NOT EQUIVALENT
- (d) If  $x, y \in A$  and  $f(x) = f(y)$ , then  $x = y$ .  
EQUIVALENT
- (e) If  $x, y \in A$  and  $f(x) \neq f(y)$ , then  $x \neq y$ .  
NOT EQUIVALENT
- (f) For each  $b \in B$  there is a unique  $a \in A$  such that  $f(a) = b$ .  
NOT EQUIVALENT

**Problem 3.** (4 points) Give a useful negation to the following statement.  
(By a useful negation, I mean you should simplify your statement as much as possible.)

“There exists  $a \in A$  such that for all  $b \in B$ ,  $a^2 = b^2$  implies  $f(a) = f(b)$ .”

ANSWER:

“For all  $a \in A$  there exists  $b \in B$  such that  $a^2 = b^2$  and  $f(a) \neq f(b)$ .”

**Problem 4.** (5 points) Let

$$A = \{1, 2, 3, \{4, 5\}, \square, \star\} \quad \text{and} \quad B = \{3, 4, 5, \star, \mathbb{N}\}.$$

Compute the following.

(a)  $A \cap B = \{3, \star\}$

(b)  $A \cup B = \{1, 2, 3, 4, 5, \{4, 5\}, \square, \star, \mathbb{N}\}$

(c)  $A - B = \{1, 2, \{4, 5\}, \square\}$

(d)  $A \cap \emptyset = \emptyset$

(e)  $B \cap \mathbb{N} = \{3, 4, 5\}$

**Problem 5.** (12 points) Using the symbols

$$0, 1, 2, 3, \dots, \aleph_0, c$$

state the cardinality of each of the following sets.

(a)  $\{5, 6, 7\}$       ANSWER: 3

(b)  $\emptyset$       ANSWER: 0

(c)  $\{2, \mathbb{N}, \{7, 8\}, a, b, \star\}$       ANSWER: 6

(d)  $\mathbb{N}$       ANSWER:  $\aleph_0$

(e)  $\mathbb{N} \times \mathbb{N}$       ANSWER:  $\aleph_0$

(f)  $\mathbb{Q}$       ANSWER:  $\aleph_0$

(g)  $\mathbb{R}$       ANSWER:  $c$

(h)  $\mathbb{C}$       ANSWER:  $c$

(i) The interval  $[3, 7)$       ANSWER:  $c$

(j) The interval  $(2, \infty)$       ANSWER:  $c$

(k)  $[3, 7) \cap \mathbb{Q}$       ANSWER:  $\aleph_0$

(l)  $\bigcup_{n=1}^{\infty} \left\{ \frac{1}{n^k} : k \in \mathbb{N} \right\}$       ANSWER:  $\aleph_0$

**Problem 6.** (5 points) Write a short response (1–3 paragraphs) answering the question:

“What is Mathematics, and what is it that mathematicians do?”

(There are many ways to answer this question, and I'm not looking for anything in particular, so feel free to express any ideas or opinions you have. Full credit will be awarded to any response that is thoughtful and provides more than just a superficial answer to the question.)

ANSWERS MAY VARY.