Assignment 7

The following problems are due at the beginning of class on Monday, Mar. 27.

Problem 1: Let X be a set and let $\mathcal{P}(X)$ be ordered by inclusion. If $\mathcal{B}$ is a nonempty subset of $\mathcal{P}(X)$, prove that $\sup(\mathcal{B}) = \bigcup_{B \in \mathcal{B}} B$ and $\inf(\mathcal{B}) = \bigcap_{B \in \mathcal{B}} B$.

Problem 2:

(a) Consider $\mathbb{N} \times \mathbb{N}$ with the dictionary order. Which elements of $\mathbb{N} \times \mathbb{N}$ have an immediate predecessor? Is there a smallest element of $\mathbb{N} \times \mathbb{N}$?

(b) Consider $\mathbb{N} \times \mathbb{N}$ with the partial order given by $(x_0, y_0) \leq (x_1, y_1)$ if and only if either $x_0 - y_0 < x_1 - y_1$, or $x_0 - y_0 = x_1 - y_1$ and $y_0 \leq y_1$. Which elements of $\mathbb{N} \times \mathbb{N}$ have an immediate predecessor? Is there a smallest element of $\mathbb{N} \times \mathbb{N}$?

(c) Consider $\mathbb{N} \times \mathbb{N}$ with the partial order given by $(x_0, y_0) \leq (x_1, y_1)$ if and only if either $x_0 + y_0 < x_1 + y_1$, or $x_0 + y_0 = x_1 + y_1$ and $y_0 \leq y_1$. Which elements of $\mathbb{N} \times \mathbb{N}$ have an immediate predecessor? Is there a smallest element of $\mathbb{N} \times \mathbb{N}$?

Problem 3: Define a relation R on points in the plane by $(x_0, y_0)R(x_1, y_1)$ if and only if $y_0 - x_0^2 = y_1 - x_1^2$. Prove that this is an equivalence relation, and describe the equivalence classes.