

Extra Cardinality Problems

The following problems are good preparation for your final exam.

1. For each of the following sets, identify whether the set is **finite**, **denumerable**, or **uncountable**. Explain why your answer is correct,

(a) The set F of all five letter words in the English language.

(b) The set $P = \{2^k : k \in \mathbb{N}\}$.

(c) The set \mathbb{D} of *dyadic rationals*, which is defined by

$$\mathbb{D} = \left\{ \frac{p}{q} \in \mathbb{Q} : p \in \mathbb{Z} \text{ and } q = 2^k \text{ for some } k \in \mathbb{Z} \right\}.$$

(d) The set \mathbb{I} of *irrational numbers* defined by $\mathbb{I} = \mathbb{R} - \mathbb{Q}$.

(e) The set X of all real roots of the polynomial $3x^3 - 2x^2 - 7x + 5$.

(f) The set \mathbb{P}_3 of polynomials of degree three with integer coefficients.

(g) The set R_3 of all real roots of polynomials of degree 3 with integer coefficients.

(h) The set \mathbb{A} of algebraic numbers, which is defined by

$$\mathbb{A} = \{x \in \mathbb{R} : x \text{ is the root of a polynomial with integer coefficients}\}.$$

(i) The set \mathbb{T} of transcendental numbers defined by $\mathbb{T} = \mathbb{R} - \mathbb{A}$, where \mathbb{A} is the set of algebraic numbers.

(j) The set of real numbers $[-3, -2) \cup \{-1\} \cup [0, 12)$.

(k) The set $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$.

(l) The set $\mathbb{R} \times \mathbb{N}$.