White Version

1. (16 pts.)

$$A = 6\ell^2, \qquad \frac{dA}{dt} = 36 \text{ in}^2/\text{h}$$

$$\frac{dA}{dt} = 12\ell\frac{d\ell}{dt} \qquad \text{hence} \qquad \frac{\mathrm{d}\ell}{\mathrm{dt}} = \frac{1}{12\ell}\frac{\mathrm{dA}}{\mathrm{dt}}$$

For $\ell = 10$ this gives $\frac{d\ell}{dt} = \frac{3}{10}$ in/h.

2. (16 pts.)

$f(x+h) \approx f(x) + f'(x)h$ for $h \approx 0$.

$f'(x) = \dfrac{2x+1}{2\sqrt{x^2+x+3}}$

Therefore $f(2.3) \approx f(2) + f'(2)(2.3-2) = 3 + \frac{5}{6} \cdot 0.3 = 3.25$

The precise value is $f(2.3) = 3.2542\ldots$

3. (16 pts.)

At $x = -3$, $x = 4$ you can use either the first derivative test or the second derivative test.

At $x = 0$ you have to use the first derivative test (the second derivative test is inconclusive because $f''(0) = 0$).

Conclusion: $x = -3$ local maximum

$x = 4$ local minimum

$x = 0$ neither local maximum nor local minimum

4. (26 pts.)

cost $= 5x^2 + 3 \cdot 4xy$, volume $= x^2y$

Hence: given: $x \geq 0$, $y \geq 0$ such that $5x^2 + 3 \cdot 4xy = 45$, want: maximum value of $V = x^2y$.

Reduce to one variable: $y = \dfrac{45-5x^2}{12x}$, $\quad V(x) = x^2y = \dfrac{1}{12}(45x - 5x^3)$.

Find domain: $y \geq 0$ implies $x \neq 0$, $x \leq 3$, hence domain is $x \in (0,3]$. Note that we could include $x = 0$ in the domain: $V(0)$ is defined, and since $V(0) = 0$, $x = 0$ will not give the maximum anyway. The advantage is that now the domain is a closed and bounded interval.

The problem therefore becomes:

Find maximum of $V(x) = \frac{1}{12}(45x - 5x^3)$ for $x \in (0,3]$ (or $x \in [0,3]$).

Critical points: **interior** either $V'(x)$ DNE (none)

or $V'(x) = 0$ ($x^2 = 3$, hence $x = \sqrt{3} \in [0,3]$);

endpoints $x = 0$ and $x = 3$.

Test: $V(\sqrt{3}) = 5\sqrt{3}/2$ and $V(0) = V(3) = 0$ (if domain is $(0,3]$, compute $\lim_{x\to 0^+} V(x) = 0$).

Conclusion: maximum at $x = \sqrt{3}$, for which $y = \dfrac{45-5x^2}{12x} = \dfrac{5}{2\sqrt{3}}$, and the volume is $5\sqrt{3}/2$ cubic feet (dimensions of x and y in feet).

5. (26 pts.)

It is easier to differentiate the function in its first form. Combine the terms afterward.

$$f(x) = \frac{1}{x} + \frac{2}{x^2}, \qquad f'(x) = \frac{-x-4}{x^3}, \qquad f''(x) = \frac{2(x+6)}{x^4}.$$

$$\lim_{x\to\pm\infty} f(x) = 0, \qquad \lim_{x\to 0} f(x) = \infty$$

I am planing to post the graph too, but was not able yet to produce it in electronic form!

asymptotes: horizontal $y = 0$ at $x \to \pm\infty$ vertical $x = 0$

critical points: $x = -4$ inflection points: $x = -6$

intervals where f is: increasing $[-4, 0)$ decreasing $(-\infty, -4], (0, \infty)$

intervals where f is: concave up $[-6, 0), (0, \infty)$ concave down $(-\infty, -6]$

local extrema: minima $x = -4$ maxima none

absolute extrema: minima $x = -4$ maxima none