Math 2431

Here are some of the topics discussed so far:

- vectors in $\mathbb{R}^n$, matrices
  - addition, multiplication by a scalar
  - product of matrices, and matrix $\times$ vector; properties (linear maps = matrix maps)
  - norm and dot product of vectors, angles and areas
- systems of linear equations: $Ax = b$
  - row echelon form via Gauss elimination
  - reduced row echelon form is unique
  - finding solutions (a basis of solutions if $b = 0$)
  - rank
  - consistent and inconsistent systems
  - if consistent: # of param's = # unknowns $-$ rank A
  - if $b = 0$, dimension of solution space = # unknowns $-$ rank A
  - superposition (§3.4)
  - solutions for low-dimensional systems
- matrices, linearity, inverses
  - linear maps (= matrix maps)
  - linear maps in $\mathbb{R}^2$
  - superposition (§3.4, §4.7)
  - computing inverses (via row reduction)
  - solving $Ax = b$ with A^{-1}
  - 2 x 2 determinants and inverses; $\text{area}(A(P)) = |\det(A)|\,\text{area}(P)$
- systems of ODE's
  - one ODE, initial value problems ($x' = f(x,t),\ x(t_0) = x_0$)
  - $x' = \lambda x \implies x(t) = x_0 e^{\lambda t}$
  - graphic representations: phase-space and time-series portraits
  - autonomous ODE's ($x' = g(x)$); equilibria ($g(x) = 0$) and stability of hyperbolic equilibria ($g'(x_0) \neq 0$)
  - 2 x 2 systems (see much more in Chapter 6)
  - solving 2 x 2 systems if eigenvalues are <u>real and distinct</u>; sinks, sources, saddles
- vector spaces
  - definition
  - subspaces (V vector space, $W \subset V$; W subspace $\iff$ W closed under addition and multiplication by scalars)
    e.g.: $\text{null}(A) = \{x \in \mathbb{R}^n \mid Ax = 0\}$, $\{X \in (\mathcal{C}^1)^n \mid X' = CX\}$
  - span of a family of vectors

- spanning sets
- linear independence
- dimension, bases
  Theorem 5.5.3, Corollary 5.6.7
- dimension of a span and of a null-space:
  $\dim(\text{span}\{w_1, \ldots, w_k\}) = \text{rank}(M^t) = \text{rank}(M),\ M = (w_1|\ldots|w_k)$
  $\dim(\text{null}(A)) + \text{rank}(A) =$ # of columns (i.e., # of variables)