

Homework 10, last part

Posted April 21, 2009

Due date: see web-site

Consider the manifold with boundary

$$M = \{z^2 = x^2 + y^2, 1 \leq z \leq 2\} \subset \mathbb{R}^3$$

oriented with the outward normal (i.e., pointing away from the z -axis; draw a figure). Consider on M the 1-form

$$\eta = i^*\omega, \quad \omega(x, y, z) = xdy$$

where $i : M \hookrightarrow \mathbb{R}^3$ is the inclusion. Check Stokes' Theorem, that is, the equality

$$\int_M d\eta = \int_{\partial M} \eta$$

where on ∂M the orientation is the one induced from M .

Hint: both integrals above are equal to -3π .

If you choose a convenient parametrization, you could decide whether it is correctly oriented by using the picture (instead of doing a computation – either approach is fine). Recall the rules we had in Calculus III about orienting the boundary – and double-check that they indeed are correct.