

Homework 5, first part

Posted March 5, 2009, corrections etc. March 13

Due date: extended to Tuesday, March 24 (originally Thursday, March 12, 2009)

Check the formulas given in class, where $g \in G = \text{GL}(n)$, $e = I$ and $A, B \in T_e G$, $f \in C^\infty(G, \mathbb{R})$:

$$(a) \quad \tilde{A}\Big|_g (f) = \frac{d}{dt}\Big|_{t=0} f(g(I + tA));$$

$$(b) \quad \tilde{A} \circ \tilde{B}(f)\Big|_e = \frac{d}{ds}\Big|_{s=0} \frac{d}{dt}\Big|_{t=0} f((I + sA)(I + tB)) = D_e^2 f(A, B) + D_e f(AB);$$

Here $\tilde{A} \circ \tilde{B}(f)$ means $\tilde{A}(\tilde{B}(f))$ where $\tilde{A}, \tilde{B} : C^\infty(M, \mathbb{R}) \rightarrow C^\infty(M, \mathbb{R})$.

(c) $f \mapsto D_e f(X)$ corresponds to $X \in T_e G$.