Here are formulas related to the stereographic charts on $S^2 \subset \mathbb{R}^3$.

$$\varphi_+ : S^2 \setminus \{(0,0,1)\} \to \mathbb{R}^2, \quad \varphi_+(x,y,z) = \left(\frac{x}{1-z}, \frac{y}{1-z}\right)$$

$$\varphi_- : S^2\setminus\{(0,0,-1)\} \to \mathbb{R}^2, \quad \varphi_-(x,y,z) = \left(\frac{x}{1+z}, \frac{y}{1+z}\right) = -\varphi_+(-(x,y,x))$$

$$\varphi_\pm^{-1}(u,v) = \left(\frac{2u}{u^2+v^2+1}, \frac{2v}{u^2+v^2+1}, \pm\frac{u^2+v^2-1}{u^2+v^2+1}\right)$$

$$\varphi_+ \circ \varphi_-^{-1}(u,v) = \varphi_- \circ \varphi_+^{-1}(u,v) = \left(\frac{u}{u^2+v^2}, \frac{v}{u^2+v^2}\right)$$

The derivative of the coordinate change $\varphi_+ \circ \varphi_-^{-1}$:

$$D[\varphi_+ \circ \varphi_-^{-1}] = \begin{pmatrix} \dfrac{v^2-u^2}{(u^2+v^2)^2} & -\dfrac{2uv}{(u^2+v^2)^2} \\ -\dfrac{2uv}{(u^2+v^2)^2} & \dfrac{u^2-v^2}{(u^2+v^2)^2} \end{pmatrix} \tag{1}$$

and evaluated at $(u,v) = \varphi_-(x,y,z)$

$$D[\varphi_+ \circ \varphi_-^{-1}]|_{(u,v)=\varphi_-(x,y,z)} = \begin{pmatrix} \dfrac{y^2-x^2}{(1+z)^2} & -\dfrac{2xy}{(1+z)^2} \\ -\dfrac{2xy}{(1+z)^2} & \dfrac{x^2-y^2}{(1+z)^2} \end{pmatrix} \tag{2}$$

For $\varphi_- \circ \varphi_+^{-1}$, we have to evaluate (1) at $(u,v) = \varphi_+(x,y,z)$; the only change in (2) is that $(1+z)$ becomes $(1-z)$.

A few more derivatives:

$$\frac{\partial}{\partial u}\left(\frac{u}{1+u^2+v^2}\right) = \frac{1-u^2+v^2}{(1+u^2+v^2)^2}$$

$$\frac{\partial}{\partial v}\left(\frac{u}{1+u^2+v^2}\right) = -\frac{2uv}{(1+u^2+v^2)^2}$$

$$\frac{\partial}{\partial u}\left(\frac{u^2+v^2-1}{u^2+v^2+1}\right) = \frac{4u}{(u^2+v^2+1)^2}$$