MATH 2331 FINAL EXAM NAME_______________

Review sheet ID #_______________

NO CALCULATORS!

1. Let $f(x)=\det\begin{bmatrix} 2 & 5 & 0 & 0 \\ 1 & x & 3 & 2 \\ 0 & 6 & 5 & 0 \\ 0 & 8 & 0 & 7 \end{bmatrix}$. Find $f'(x)$. 12 pts

2. A rank one 3x3 symmetric matrix has column space containing the vector (1,2,3). Find a basis and the dimension of the null space. 12 pts

3. a. Find the eigenvalues and eigenvectors of each of these matrices. Identify which are invertible and/or diagonalizable.

$A=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ 6 pts

$B=\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ 6 pts

$C=\begin{bmatrix} 2 & 5 \\ -1 & -2 \end{bmatrix}$ 6 pts

$D=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ 6 pts

$E=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ 6 pts

$F=\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix}$ 6 pts

4. a. Find the determinant of this N-shaped matrix:

$N=\begin{bmatrix} 1 & 0 & 0 & 4 \\ 2 & 1 & 0 & 3 \\ 3 & 0 & 1 & 2 \\ 4 & 0 & 0 & 1 \end{bmatrix}$ 10 pts

b. What is the rank of N-I? Find all four eigenvalues of N. 10 pts

5. For what vectors $\mathbf{b}$ does the system $\mathbf{Ax}=\mathbf{b}$ have a solution,
if $\mathbf{A}=\begin{bmatrix} 1 & 3 & 1 \\ 0 & -1 & -1 \\ 2 & 4 & 0 \end{bmatrix}$? Find an equation for $\mathbf{b}$: $c_1b_1+c_2b_2+\ldots+c_nb_n=0$

12 pts

6. a. Suppose $\mathbf{q}_1, \mathbf{q}_2, \mathbf{a}_3$ are linearly independent vectors. $\mathbf{q}_1$ and $\mathbf{q}_2$ are already orthonormal. Give a formula for a third orthonormal vector $\mathbf{q}_3$ as a linear combination of $\mathbf{q}_1, \mathbf{q}_2, \mathbf{a}_3$.

10 pts

b. Find the vector $\mathbf{q}_3$ of part (a) when

$$\mathbf{q}_1=\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{q}_2=\frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{a}_3=\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

10 pts

7. This problem uses least squares to find the line y = ax + b that best fits these 4 points in the plane:
$(x_1,y_1)=(-2,1)$, $(x_2,y_2)=(0,0)$, $(x_3,y_3)=(1,2)$, $(x_4,y_4)=(1,4)$.

a. Write down 4 equations $ax_i+b=y_i$, i = 1, 2, 3, 4, that would be true if the line actually went through all four points.

8 pts

b. Now write those four equations in the form $\mathbf{A}\begin{bmatrix} a \\ b \end{bmatrix}=\mathbf{y}$

4 pts

c. Now find $\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}$ that minimizes $\left\|\mathbf{A}\begin{bmatrix} a \\ b \end{bmatrix}-\mathbf{y}\right\|^2$.

14 pts

8. Let $f(x)=\det\begin{bmatrix} 5 & 0 & 3 \\ 2+x & 3x+1 & 4x-2 \\ 0 & 2 & 1 \end{bmatrix}$. Find f′(x).

12 pts

9. For each 2x2 matrix **A** below, draw a picture in the xy plane that shows A*house, where "house" is the set of points: $\{(0,\ 0),\ (2,\ 0),\ (2,\ 2),\ (0,\ 2),\ (1,\ 2.5)\}$ including lines for the floor, walls, ceiling, and roof, as shown below.

20 pts

a. $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

b. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

c. $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$

d. $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

10. Find the complete solution to the system:

B = $\begin{bmatrix} 1 & 2 & -1 & -5 \\ 2 & 4 & -1 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ 12

11. Find a subset of these vectors that forms a basis for the span of the vectors. Express the vectors not in the basis as combinations of the basis.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{a}_4 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{a}_5 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ 3 \\ 1 \end{bmatrix}$$ 20

12. A 3x2 matrix (3 rows, 2 columns) A has a null space spanned by $\begin{bmatrix} 3 & 4 \end{bmatrix}$. The column space is spanned by $\begin{bmatrix} 2 & 1 & 2 \end{bmatrix}$. Also

$$A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix}$$

a. Find a basis for the row space. 8 pts

b. Use the SVD to find A. 10 pts

13. Find the eigenvalues and one real eigenvector of this permutation matrix.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$ 15 pts

14. This symmetric Markov matrix has zero determinant:

$$A = \begin{bmatrix} .4 & .2 & .4 \\ .2 & .6 & .2 \\ .4 & .2 & .4 \end{bmatrix}$$

a. What are the eigenvalues of A? 10 pts

b. Find $\lim_{k\to\infty} \mathbf{A}^k \mathbf{u}_0$ with $u_0 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ 15 pts

15. $A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$

a. Find all of the eigenvalues of A. 6 pts

b. Find a complete set of unit eigenvectors of A. 8 pts

c. Find orthogonal matrices **U** and **V**, and diagonal Σ so that $\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^T$