Math 2433 **Exam 2** **Name**_____
April 3, 2009 PS #_____

NO CALCULATORS!

1. Determine which of the following vector fields is the gradient of a function $f(x,y)$. If it is, find all such functions $f(x,y)$. 18 pts

   a. $V(x,y) = \left(3x^2 + e^y\right)\mathbf{i} + \left(xe^y - 7y^3\right)\mathbf{j}$

   b. $\mathbf{W}(x,y) = \left(\sin(x) + e^y\right)\mathbf{i} + \left(\cos(x)y + e^x\right)\mathbf{j}$

2. If $f(x,y) = 3x^2 - y^2$, $\mathbf{r}(0) = (2,3)$, $\mathbf{r}'(0) = (3,5)$ and $h(t) = f(\mathbf{r}(t))$: find $h'(0)$. 9 pts

3. Find the point on the curve $y^2 - x^2 = 1$ closest to the point (0,4). 12 pts

4. a. Find an equation for the tangent plane to the surface $xy^2 + 2yz^2 = 40$, at the point (x,y,z) = (1,2,3). 8 pts

   b. Find equations for the normal line to this surface at (x,y,z) = (1,2,3). 6 pts

5. a. Find a unit vector in the direction in which the function $f(x,y) = \ln(1 + 12x + 6y) - y$ increases most rapidly, at $(x,y) = (0,0)$. 8 pts

   b. What is the directional derivative of f in this direction? 6 pts

   c. Find the directional derivative of f in the direction of $3\mathbf{i} - 4\mathbf{j}$ at $(x,y) = (0,0)$. 6 pts

6. Suppose $f(x,y) = xe^y + ye^z + ze^x$. 12 pts
   Find $f_{xx} + f_{yy} + f_{zz}$.

7. Find all of the critical points of the function

   $f(x,y) = x^3 - 27x + y^2 - 6y$

   and determine whether each critical point yields a maximum value, minimum value, or saddle point. 15 pts