Motivational Problems
Differentiation

MAT1450: Accelerated Calculus Sec 17957 Fall 2017

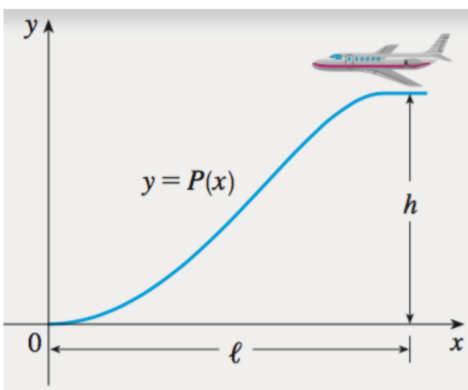
Due date: ~~Sep 07, 2017~~ Sep 14, 2017 in class - optional

1. **Plane Descent.** An approach path for an aircraft landing is shown in the figure and satisfies the following conditions:

   (i) The cruising altitude is h when descent starts at a horizontal distance l , from touchdown at the origin.

   (ii) The pilot must maintain a constant horizontal speed v throughout descent.

   (iii) The absolute value of the vertical acceleration should not exceed a constant k (which is much less than the acceleration due to gravity).



   (a) Find a cubic polynomial $P(x) = ax^3 + bx^2 + cx + d$ that satisfies condition (i) by imposing suitable conditions on $P(x)$ and $P'(x)$ at the start of descent and at touchdown.

   (b) Use conditions (ii) and (iii) to show that

$$\frac{6hv^2}{l^2} \leq k.$$

   (c) Suppose that an airline decides not to allow vertical acceleration of a plane to exceed $k = 860 mi/h^2$. If the cruising altitude of a plane is 35,000 ft and the speed is 300 mi/h, how far away from the airport should the pilot start descent?

2. **Cardiac Output.** In the late 1860s, Adolf Fick, a professor of physiology in the Faculty of Medicine in Würzberg, Germany, developed one of the methods we use today for measuring how much blood your heart pumps in a minute. Your cardiac output as you read this sentence is probably about 7 L/min. At rest it is likely to be a bit under 6 L/min. If you are a trained marathon runner running a marathon, your cardiac output can be as high as 30 L/min.

   Your cardiac output can be calculated with the formula

$$y = \frac{Q}{D},$$

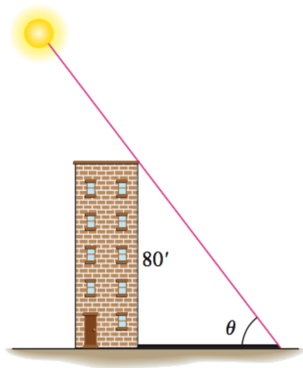
   where Q is the number of milliliters of CO_2 you exhale in a minute and D is the difference between the CO_2 concentration (ml/L) in the blood pumped to the lungs and the CO_2 concentration in the blood returning from the lungs. With $Q = 233$ ml/min and $D = 97 - 56 = 41$ ml/L,

$$y = \frac{233}{41} \sim 5.68 L/min,$$

   fairly close to the 6 L/min that most people have at basal (resting) conditions. (Data courtesy of J. Kenneth Herd, M.D., Quillan Col- lege of Medicine, East Tennessee State University.)

   Suppose that when Q = 233 and D = 41, we also know that D is decreasing at the rate of 2 units a minute but that Q remains unchanged. What is happening to the cardiac output?

3. **Building Shadow.** On a morning of a day when the sun will pass directly overhead, the shadow of an 80-ft building on level ground is 60 ft long.



   At the moment in question, the angle θ the sun makes with the ground is increasing at the rate of $0.27°$/min. At what rate is the shadow decreasing? (Remember to use radians. Express your answer in inches per minute, to the nearest tenth.)

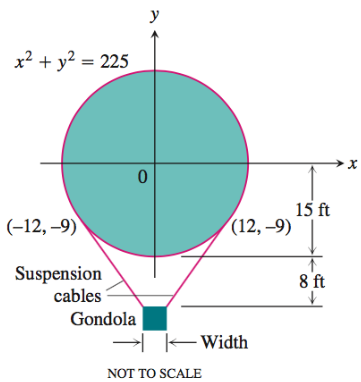
4. **Bus Fare.** A bus will hold 60 people. The number x of people per trip who use the bus is related to the fare charged (p dollars) by the law

$$p = [3 - (x/40)]^2.$$

   Write an expression for the total revenue $r(x)$ per trip received by the bus company. What number of people per trip will make the marginal revenue dr/dx equal to zero? What is the corresponding fare? (This fare is the one that maximizes the revenue, so the bus company should probably rethink its fare policy.)

5. **Clearing the intersection.** You are coming up to the intersection, where a car is waiting on a red light. In 4 seconds it will be green. You cannot pass the car, but want to get to your destination as fast as possible. What should your speed be when a red light is 72 meters away? The waiting car will accelerate at 3 m/sec^2. First explain your strategy, then find the speed.

6. **Flying Air Baloon.** The designer of a 30-ft-diameter spherical hot air balloon wants to suspend the gondola 8 ft below the bottom of the balloon with cables tangent to the surface of the balloon, as shown. Two of the cables are shown running from the top edges of the gondola to their points of tangency, $(-12,-9)$ and $(12,-9)$. How wide should the gondola be?



7. **Surgery.** A typical volume of blood in the human body is about 5 L. A certain percentage of that volume (called the hematocrit) consists of red blood cells (RBCs); typically the hematocrit is about 45% in males. Suppose that a surgery takes four hours and a male patient bleeds 2.5 L of blood. During surgery the patient's blood volume is maintained at 5 L by injection of saline solution, which mixes quickly with the blood but dilutes it so that the hematocrit decreases as time passes.

   (a) Assuming that the rate of RBC loss is proportional to the volume of RBCs, determine the patient's volume of RBCs by the end of the operation.

   (b) A procedure called acute normovolemic hemodilution (ANH) has been developed to minimize RBC loss during surgery. In this procedure blood is extracted from the patient before the operation and replaced with saline solution. This dilutes the patient's blood, resulting in fewer RBCs being lost during the bleeding. The extracted blood is then returned to the patient after surgery. Only a certain amount of blood can be extracted, however, because the RBC concentration can never be allowed to drop below 25% during surgery. What is the maximum amount of blood that can be extracted in the ANH procedure for the surgery described in this project?

   (c) What is the RBC loss without the ANH procedure? What is the loss if the procedure is carried out with the volume calculated in Problem 2?

8. **Fight or flight?** A thief 40 meters away runs toward you at 8 meters per second. You start running away from him with a constant acceleration. What is the smallest acceleration that keeps you in front of the thief? [The formula of distance s in terms of acceleration a is $s = at^2/2$.]

9. **Fish Farm.** In a fish farm, a population of fish is introduced into a pond and harvested regularly. A model for the rate of change of the fish population is given by the equation

$$\frac{dP}{dt} = r_0 \left(1 - \frac{P(t)}{P_c}\right) P(t) - \beta P(t),$$

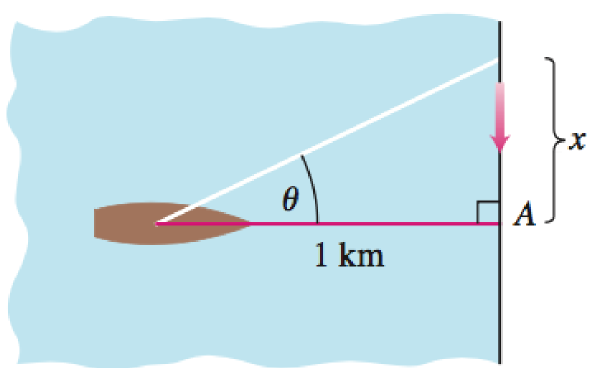
where r_0 is the birth rate of the fish, P_c is the maximum population that the pond can sustain (called the carrying capacity), and β is the percentage of the population that is harvested.

   (a) What value of dP/dt corresponds to a stable population?

   (b) If the pond can sustain 10,000 fish, the birth rate is 5%, and the harvesting rate is 4%, find the stable population level.

   (c) What happens if β is raised to 5%?

10. **Harbour Patrol.** The figure below shows a harbour patrol boat 1 km off-shore, sweeping the shore with a searchlight.



The light turns at a constant rate, $d\theta/dt = -0.6$ rad/sec.

   (a) How fast is the light moving along the shore when it reaches point A?

   (b) How many revolutions per minute is 0.6 rad/sec?