Midterm 1

MAT1450   Sec: 17957 (Dr. Vershynina)   Fall 2017

September 28, 2017

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Do not turn the page until instructed to do so.

No books, notes, calculators, laptops, cell phones or any other help is permitted on the test.

Show all work, points will be deducted if work is sloppy or not shown. Write your arguments in a logical, well-organized and clear way.

If you need more paper use the back sides of the pages or ask for additional scratch paper. Do not forget to write your name, ID and problem number on the scratch paper and attach it to your exam.

The duration of the exam is 80 min.

You are expected to adhere to the Academic UH Honesty Policy.

Take a deep breath, and good luck!

Do not write anything on this page!

Problem	Score
1 [6 points]	
2 [10 points]	
3 [10 points]	
4 [14 points]	
5 [15 points]	
6 [10 points]	
Total out of 65	
Percentage	

1. [6 points] Differentiate

   (a) $y = \ln(\sin^2 \theta)$

   (b) $r = \sqrt{\frac{t^2 + t}{t^2}}$

   (c) $w = \tan^{-1}(e^{2x})$

2. (a) [4 points] State without proof the following limits

   i. $\lim_{x\to\infty} \dfrac{x^7 - 1}{\sqrt{7x^7 + x^5 - 3x^2 + 1}} =$

   ii. $\lim_{x\to 0} \dfrac{3^x - 1}{2^x - 1} =$

   (b) [6 points] Suppose that a function $f(x)$ such that $f'(x) < 1$ for all x on its domain. Show that $|f(x) - f(y)| < |x - y|$ for all points x, y on the domain of f.

3. [10 points] A particle with coordinates $(x(t), y(t))$ is constrained to move on the ellipse

$$2y^2 + x^2 = 3.$$

Assume that $\dfrac{dx}{dt} \neq 0$ and $\dfrac{dy}{dt} \neq 0$. Find the two points on the ellipse where $\dfrac{dx}{dt} = \dfrac{dy}{dt}$.

4. (a) [7 points] Find the point on the curve $y = \sqrt{x}$ closest to the point $(2, 0)$.

(b) [7 points] If 100 square centimeters of material is available to form the sides and base of a box with a square base and an open top, what is the largest possible volume of the box? (The box need not be a cubic box).

5. The rate of decay of uranium of mass m with respect to time t is proportional to m.

    (a) [4 points] Write down an equation expressing the previous statement.

    (b) [6 points] Suppose that 250 grams of uranium has decayed to 50 grams over a period of 1 year. Find the half-life of uranium.

    (c) [5 points] Find the amount of uranium remaining after another 1 year passed.

6. For the function $f(x) = x^3 - 3x^2 - x + 3$ find the set of x values for which:

   (a) [2 points] f is increasing

   (b) [2 points] f is decreasing

   (c) [2 points] the graph of f is concave up

(d) [2 points] the graph of f is concave down

(e) [2 points] Draw a rough sketch of the graph of f