Brief Description of the Course
High-Dimensional Measures and Geometry
MATH 6397, Spring 2010

Content: This course covers many aspects of the phenomenon that functions of small oscillation become nearly constant in high-dimensional spaces. This principle, developed by Milman for Banach spaces, has applications in geometry, probability and statistics, functional analysis, discrete mathematics and even in complexity theory.

In an introductory part, some interesting features of Boolean cubes and Euclidean balls in high dimensions will be discussed. We will also see how integration with respect to a suitable Gaussian measure and with respect to the surface measure of the sphere are more and more indistinguishable in high dimensions.

The probabilistic aspects of the concentration of measure phenomenon start with the traditional laws of large numbers for independent random variables and random processes. When reformulated in a geometric fashion, this allows to find more general versions of this phenomenon. When used with randomization techniques, this leads to a result on metric embeddings: the existence of approximately round sections of any convex body. We will also study a recent application of embeddings in compressive sensing.

In the final part of the course, a more abstract approach to measure concentration is derived from semigroup tools, which yields concentration via the dimension-independent logarithmic Sobolev inequality. This will be studied on Euclidean spaces and on graphs with suitable connectivity properties.

Prerequisites: Graduate standing, a course on probability and a graduate-level course on measure theory.

Text: Michel Ledoux, The Concentration of Measure Phenomenon, AMS 2001 ($65 or, for AMS members, $52).

Grade: The grade will be based on notes taken and typeset by the students. If the enrollment number permits, students may review and present selected research papers related to the course topics during the last 1-2 weeks.