Syllabus

High-Dimensional Measures and Geometry
MATH 6397, Fall 2020

Content: This course covers many aspects of the phenomenon that functions of small oscillation become nearly constant in high-dimensional spaces. This principle, developed by Milman for Banach spaces, has applications in geometry, probability and statistics, functional analysis, discrete mathematics and even in quantum information theory and complexity theory.
In an introductory part, some interesting features of Boolean cubes and Euclidean balls in high dimensions will be discussed. We will also see how integration with respect to a suitable Gaussian measure and with respect to the surface measure of the sphere are more and more indistinguishable in high dimensions.
The probabilistic aspects of the concentration of measure phenomenon start with the traditional laws of large numbers for independent random variables and random processes. When reformulated in a geometric fashion, this allows to find more general versions of this phenomenon. We will even establish a version of the central limit theorem for matrices!
In the final part of the course, we will discuss applications ranging from compressed sensing and machine learning to quantum information theory.

Prerequisites: Graduate standing, a course on Probability and a graduate-level course on Analysis.

Text (recomm.): The materials will be collected from the following recommended monographs. Roman Vershynin, High-Dimensional Probability: An Introduction with Applications in Data Science (Cambridge Series in Statistical and Probabilistic Mathematics), Cambridge University Press, 2018. Michel Ledoux, The Concentration of Measure Phenomenon, AMS, 2001. Holgert Rauhut and Simon Foucart, A Mathematical Introduction to Compressive Sensing, Birkhäuser, 2013.

Topics papers: "An Elementary Proof of a Theorem of Johnson and Lindenstrauss" by Sanjouy Dasgupta and Anupam Gupta, "A Simple Proof of the Restricted Isometry Property for Random Matrices" by Richard Baraniuk, Mark Davenport, Ronald DeVore, and Michael Wakin, "Decoding by Linear Programming" by Emmanuel Candès and Terry Tao, "The restricted isometry property and its implications for compressed sensing" by Emmanuel Candès, "Hastings' additivity counterexample via Dvoretzky's theorem" by Guillaume Aubrun, Stanislaw Szarek, and Elisabeth Werner. Other papers may be included by suggestion.

Grade: The grade will be based on presentations of topics papers by the students during the second half of the course.

Face Covering: To reduce the spread of COVID-19, the University requires face coverings on campus including classrooms for both faculty and students. Face coverings must cover your mouth and nose and be worn throughout the class session. A mask with a valve is not considered an adequate face covering and should not be used, as it can expel exhaled air, increasing the risk to others. Eating or drinking during class is discouraged and is not an excuse for removing the face covering for any extended length of time. For additional information on the use of face coverings, please see Face Covering FAQs. Failure to comply with the requirement to wear a face covering in class will result in your being asked to leave the classroom immediately and a disciplinary referral through the Dean of Students Office. Requests for accommodations relating to the face covering policy may be directed to the Center for Students with DisABILITIES (CSD).

Absences: Regular class attendance, participation, and engagement in coursework are important contributors to student success. Absences may be excused as provided in the University of Houston Undergraduate Excused Absence Policy and Graduate Excused Absence Policy for reasons including: medical illness of student or close relative, death of a close family member, legal or government proceeding that a student is obligated to attend, recognized professional and educational activities where the student is presenting, and University-sponsored activity or athletic competition. Additional policies address absences related to military service, religious holy days, pregnancy and related conditions, and disability.

Recording Classes: Students may not record all or part of class, livestream all or part of class, or make/distribute screen captures, without advanced written consent of the instructor. If you have or think you may have a disability such that you need to record class-related activities, please contact the Center for Students with DisABILITIES. If you have an accommodation to record class-related activities, those recordings may not be shared with any other student, whether in this course or not, or with any other person or on any other platform. Classes may be recorded by the instructor. Students may use instructor?s recordings for their own studying and notetaking. Instructor?s recordings are not authorized to be shared with anyone without the prior written approval of the instructor. Failure to comply with requirements regarding recordings will result in a disciplinary referral to the Dean of Students Office and may result in disciplinary action.

Syllabus Changes: Due to the changing nature of the COVID-19 pandemic, please note that the instructor may need to make modifications to the course syllabus and may do so at any time. Notice of such changes will be announced as quickly as possible through (specify how students will be notified of changes).