

Math 4310/6317, Fall 2011
Problem Set 2, due Thursday, Sep 8

Problem 1. Using the rules of expectations prove that $\text{Var}(X) = E[X^2] - E[X]^2$ starting from the definition $\text{Var}(X) = E[(X - \mu)^2]$.

Problem 2. Suppose that a random variable X has a density of the form

$$f(x) = \begin{cases} cx^k & , 0 \leq x \leq 1 \\ 0 & , \text{else} \end{cases}$$

for some constant $k \geq 0$.

- Find c .
 - Derive the distribution function for the density f .
 - Derive a formula for the p^{th} quantile from f .
 - Let $0 \leq a < b \leq 1$. Compute $P(a < X < b)$.
 - Compute the mean and the variance of X .
- Problem 3. Let $g(x) = sf_1(x) + (1-s)f_2(x)$ where $0 \leq s \leq 1$ and the densities f_1 and f_2 are associated with means and variances μ_1, σ_1^2 and μ_2, σ_2^2 , respectively. Show that g is a valid density, and compute its associated mean and variance.
- Problem 4. You are playing a game with a friend where you flip a coin and if it comes up heads you give him a dollar and if it comes up tails she gives you a dollar. You play the game ten times.
- What is the expected total earnings for you? (Show your work; state your assumptions.)
 - What is the variance of your total earnings? (Show your work; state your assumptions.)
 - Suppose that the coin is biased and you have a .4 chance of winning for each flip. repeat the calculations in parts a and b
- Problem 5. Assume that swimming in the bayou is associated with $1/500$ probability of getting an infection. How many times would an uninfected person have to have repeat a swim in the bayou to have a 50% probability of obtaining an infection? State the assumptions of your calculations.