MATH 4331

Introduction to Real Analysis

Fall 2013

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 1, due Thursday, September 5, 10am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 0

Prove that if a sequence $\{p_1, p_2, \dots\}$ converges to $\lim_{n\to\infty} p_n = p$, then it is bounded, that is, there exists a $L \geq 0$ such that for all $n \in \mathbb{N}$, $|p_n| \leq L$. Hint: Start with: Convergence of $\{p_n\}_{n\in\mathbb{N}}$ means that for every $\epsilon > 0$, ... Now *choose any* $\epsilon > 0$, then split the set $\mathbb{N}$ into two subsets and show the boundedness on each of those subsets.

Problem 1

Prove that if we define for $x, y \in \mathbb{R}$, $d(x, y) = |x^2 - y^2|$ then $(\mathbb{R}, d)$ **is not** a metric space.

Problem 2

Prove that if we define for $x, y \in \mathbb{R}$, $d(x, y) = |x - y| + |x^2 - y^2|$ then $(\mathbb{R}, d)$ **is** a metric space. Mention all of the properties that are required, even if some may seem obvious to you.

Problem 3

Prove that $(\mathbb{R}^2, d)$ is a metric space if we define that for any two points $(x, y), (x', y') \in \mathbb{R}^2$, their distance is

$$d((x, y), (x', y')) = \begin{cases} |y| + |y'| + |x - x'| & \text{if } x \neq x', \\ |y - y'| & \text{if } x = x'. \end{cases}$$