MATH 4331

Introduction to Real Analysis

Fall 2013

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 2, due Thursday, September 12, 10am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Prove that if $\mathbb{R}^2$ is equipped with the usual Euclidean metric, then the set $A = \{(x_1, x_2) : x_1+x_2 > 0\}$ is open.

Problem 2

Prove that if $\mathbb{R}^2$ is equipped with the max-metric

$$d_\infty((x_1, x_2), (y_1, y_2)) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$$

then the disk

$$D = \{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 < 1\}$$

is an open set.

Problem 3

Let $X = C([0, 1])$ be the space of continuous real-valued functions on $[0, 1]$ with the max-metric

$$d_\infty(f, g) = \max\{|f(t) - g(t)| : 0 \leq t \leq 1\} .$$

Prove that the set $P = \{f \in C([0, 1]) : f(t) > 0 \text{ for all } 0 \leq t \leq 1\}$ is open.

Problem 4

Prove that on $\mathbb{R}^2$, the two metrics d_1 and d_∞ are uniformly equivalent. Hint: You may quote results from class relating d_1 and d as well as d and d_∞ without proof.