

MATH 4331
Introduction to Real Analysis
Fall 2013

First name: _____ Last name: _____

Points:

Assignment 3, due Thursday, September 19, 10am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let \mathbb{R}^2 be equipped with the usual Euclidean metric. Show that the set

$$S = \{(x_1, x_2) : 0 \leq x_2 \leq x_1\}$$

is closed.

Problem 2

Show that every open set A in a metric space (X, d) is the union of closed sets.

Problem 3

Prove with the same methods as in class that the sequence $p_n = n \sin \frac{1}{n}$ in \mathbb{R} with the usual metric converges to $\lim_{n \rightarrow \infty} p_n = 1$.

Problem 4

Let (X, d) be a metric space and $A \subset X$. Let E be the set of all $p \in X$ for which there is a sequence $\{p_n\}_{n \in \mathbb{N}}$ with $p_n \in A$ for each $n \in \mathbb{N}$ and $\lim_{n \rightarrow \infty} p_n = p$. Show that E is the closure of A .