MATH 4331

Introduction to Real Analysis

Fall 2013

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 4, due Thursday, October 3, 10am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let $\mathbb{R}^2$ be equipped with the usual Euclidean metric. Show that the open ball $B(0,1) = \{(x_1, x_2) : x_1^2 + x_2^2 < 1\}$ has the boundary

$$\partial B(0,1) = \{(x_1, x_2) : x_1^2 + x_2^2 = 1\} .$$

Problem 2

Let (X, d) be a discrete metric space. For any $A \subset X$, describe the closure, the interior and the boundary of A.

Problem 3

Let X be a set and let d and ρ be two uniformly equivalent metrics on X. Prove that (X, d) is a complete metric space if and only if (X, ρ) is complete.

Problem 4

Let $\mathbb{R}$ be equipped with the usual metric and $A = \{\frac{1}{n} : n \in \mathbb{N}\}$.

a. Show that A is not compact.

b. Show that $A \cup \{0\}$ is compact.