MATH 4331

Introduction to Real Analysis

Fall 2013

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 6, due Thursday, October 31, 10am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let $\mathbb{R}$ be equipped with the usual metric. Prove that $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3$ is not uniformly continuous.

Problem 2

Let (X, d) and (Y, ρ) be metric spaces. A function $f : X \to Y$ is called Lipschitz continuous provided that there is a constant $K > 0$ so that $\rho(f(p), f(q)) \leq Kd(p, q)$ for all $p, q \in X$. Prove that every Lipschitz continuous function is uniformly continuous.

Problem 3

Let (X, d) and (Y, ρ) be metric spaces. Prove that if $f : X \to Y$ is continuous, then for any set A in X with closure $\overline{A}$,

a. we have $f(\overline{A}) \subset \overline{f(A)}$

b. and this inclusion can be proper, i.e. give an example for which $f(\overline{A}) \neq \overline{f(A)}$.

Problem 4

Let (X, d) and (Y, ρ) be metric spaces. Prove that if $f : X \to Y$ is one-to-one, continuous, and onto, and if X is compact, then the inverse $f^{-1} : Y \to X$ is continuous.