Department of Mathematics University of Houston

# MATH 4331

## Introduction to Real Analysis

Fall 2015

**First name:** ____________________ **Last name:** ______________________ **Points:**

# Assignment 2, due Thursday, September 10, 2:30pm

**Please staple this cover page to your homework.** When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

## Problem 1

The interior $A^\circ$ of a set $A \subset \mathbb{R}^n$ is defined as largest open subset, or equivalently, as the set containing each point $x \in A$ for which there exists $r > 0$ such that $B_r(x) \subset A$.

Show that $A^\circ = (\overline{A'})'$, that is, the interior of $A$ is obtained by taking the complement of the closure of the complement of $A$.

## Problem 2

Suppose that $A$ and $B$ are subsets of $\mathbb{R}$.

a. Show that if $A$ and $B$ are closed, then the set $A \times B = \{(x, y) \in \mathbb{R}^2 : x \in A, y \in B\}$ is closed in $\mathbb{R}^2$.

b. Likewise, show that if $A$ and $B$ are both open, then $A \times B$ is open.

## Problem 3

A set $A$ is dense in $B$ if $B$ is contained in $\overline{A}$.

a. Show that the set of irrational numbers, $\mathbb{R} \setminus \mathbb{Q}$, is dense in $\mathbb{R}$.

b. Hence, show that $\mathbb{Q}$ has empty interior.

## Problem 4

Show that the union of finitely many compact sets $C_1$, $C_2$, …, $C_m$ in $\mathbb{R}^n$ is compact.

## Problem 5

Show that the intersection of any family of compact sets $\{C_i\}_{i \in I}$ in $\mathbb{R}^n$ is compact.