Department of Mathematics University of Houston

# MATH 4331

## Introduction to Real Analysis

Fall 2015

**First name:** _____ **Last name:** _____ **Points:**

# Assignment 4, due Thursday, September 24, 2:30pm

**Please staple this cover page to your homework.** When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

## Problem 1

Show that if $S$ is a connected set in $\mathbb{R}^n$, then so is its closure $\overline{S}$.

## Problem 2

Suppose that $A$ is a subset of $\mathbb{R}^m$ and $B$ a subset of $\mathbb{R}^n$. Show that if $A$ and $B$ are connected, then the so is the set $A \times B = \{(x, y) \in \mathbb{R}^{m+n} : x \in A, y \in B\}$.

## Problem 3

Let $S = Y \cup G$ be the union of two sets in $\mathbb{R}^2$ given by

$$Y = \{(0, t) : |t| \leq 1\}$$

and

$$G = \{(x, \sin(1/x)) : 0 < x \leq 1\}.$$

Show that $S$ is connected, but not path connected.

## Problem 4

Let $A_1 \supset A_2 \supset A_3 \supset \dots$ be a decreasing sequence of connected compact subsets of $\mathbb{R}^n$. Show that $\cap_{k \geq 1} A_k$ is connected.

## Problem 5

Give an example for a sequence of subsets $A_1 \supset A_2 \supset A_3 \supset \dots$ of $\mathbb{R}^2$ where the sets $A_k$ are closed and connected and $\cap_{k \geq 1} A_k$ is **not** connected.