Department of Mathematics University of Houston

MATH 4331

Introduction to Real Analysis

Fall 2015

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 5, due Thursday, October 1, 2:30pm

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let f and g be differentiable functions on an interval (a, b), $a < b$. If there is x_0 for which $f(x_0) = g(x_0)$ and $f(x) \leq g(x)$ for all $x \in (a, b)$, prove that $f'(x_0) = g'(x_0)$.

Problem 2

Show the product rule: If f and g are differentiable functions on an interval (a, b) and $x_0 \in (a, b)$, then $(fg)'(x_0) = f(x_0)g'(x_0) + f'(x_0)g(x_0)$.

Problem 3

If f and g are differentiable on $[a, b]$ and $f'(x) = g'(x)$ for all $a < x < b$, prove that $g(x) = f(x) + C$ for some constant $C \in \mathbb{R}$.

Problem 4

Assume f is differentiable on $[a, b]$ and $f'(a) < 0 < f'(b)$. Show the following:

(a) There are c, d with $a < c < d < b$ and $f(c) < f(a)$ as well as $f(d) < f(b)$.

(b) The minimum of f on $[a, b]$ occurs at $x_0 \in (a, b)$.

(c) Hence, there is $x_0 \in (a, b)$ with $f'(x_0) = 0$.

Use this to prove that if f is differentiable on $[a, b]$, and $f'(a) < L < f'(b)$, then there is $x_0 \in (a, b)$ with $f'(x_0) = L$.

Problem 5

If f is differentiable on $\mathbb{R}$ and f' is strictly increasing, show that f' is continuous. Hint: Prove that if a function g is strictly increasing, then either $\sup_{x<a} g(x) < \inf_{x>a} g(x)$ or g is continuous at $a \in \mathbb{R}$.