Practice Exam 1 – Math 4331/6312
October, 2013

First name: ________________ Last name: ________________ Last 4 digits of Student ID: _______

1 True-False Problems

Put a T in the box beside each statement that is true, an F if the statement is false.

- ☐ Every convergent sequence in $\mathbb{R}^n$ is bounded.
- ☐ Every convergent sequence in $\mathbb{R}^n$ is Cauchy.
- ☐ If U is an open subset and C is a closed subset of $\mathbb{R}^n$, then $U \setminus C$ is an open subset of $\mathbb{R}^n$.
- ☐ The Cantor set is compact.
- ☐ If $S = [0, 1) \cup (1, 2]$, then S is a connected subset of $\mathbb{R}$.
- ☐ If $f : S \subset \mathbb{R}^m \to \mathbb{R}^n$ is continuous, then $f^{-1}(U)$ is open in $\mathbb{R}^m$ for any set U that is open in $\mathbb{R}^n$.
- ☐ If $S = \{(x, y) : 0 \leq y \leq \frac{1}{x}, x > 0\} \cup \{(0, y) : y \in \mathbb{R}\}$ then S is a closed set.
- ☐ If f is differentiable and strictly increasing on $\mathbb{R}$, then $f'(x) > 0$ for all $x \in \mathbb{R}$.

After completing this part, hand it in to obtain the remaining portion of the exam.

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In the following problems, you may quote results from class to simplify your answers. You do not need to include a proof of a statement if it was discussed in class.

2 Problem

Let $p_k = (\frac{1}{k}, \frac{1}{k^2})$, $k \in \mathbb{N}$, define a sequence in $\mathbb{R}^2$. Prove that $\lim_{k\to\infty} p_k = (0, 0)$.

3 Problem

Let $E \subset \mathbb{R}^n$. Show that x is a cluster point of E if and only if $x \in \overline{E \setminus \{x\}}$.

4 Problem

Prove that the intersection of two compact sets C_1 and C_2 in $\mathbb{R}^n$ is a compact set.

5 Problem

(a) Consider a function $f : S \subset \mathbb{R}^m \to T \subset \mathbb{R}^n$. State the definition of uniform continuity for f.

(b) Prove that if $f : S \subset \mathbb{R}^m \to T \subset \mathbb{R}^n$ and $g : T \subset \mathbb{R}^n \to \mathbb{R}$ are both uniformly continuous on their domains, then the composition $h = g \circ f, h(x) = g(f(x))$ is uniformly continuous on S.

6 Problem

Let $0 < \epsilon < 1$. Use the Mean-Value Theorem to prove that the real-valued, differentiable function

$$f(x) = x^2 \cos(\pi/x)$$

has a point $c \in (0, \epsilon)$ with $f'(c) > 1$.

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