Department of Mathematics University of Houston

MATH 4331/6312

Introduction to Real Analysis

Fall 2017

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 4, due Thursday, September 28, 10am

Please staple this cover page to your homework. Circle your course number, Math 4331 or 6312. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let f be a continuous function defined on an **open** subset S of $\mathbb{R}^n$. Prove that the set $\{(x_1, x_2, \ldots, x_n, y) : x \in S, y > f(x)\}$ is an open subset of $\mathbb{R}^{n+1}$. If useful, abbreviate $(x_1, x_2, \ldots, x_n, y) = (x, y)$.

Problem 2

Show that the function $f(x) = x^p$ on defined on $\mathbb{R}$ is not uniformly continuous if $p \in \{2, 3, 4, \ldots\}$.

Problem 3

A function $f : \mathbb{R} \to \mathbb{R}$ is called 1-periodic if $f(x) = f(x + 1)$ for each $x \in \mathbb{R}$. Show that if f is continuous, then it is uniformly continuous.

Problem 4

Prove that if S is a connected set in $\mathbb{R}^n$, then so is its closure $\overline{S}$.

Problem 5

Suppose that A is a subset of $\mathbb{R}^m$ and B a subset of $\mathbb{R}^n$. Show that if A and B are connected, then the so is the set $A \times B = \{(x, y) \in \mathbb{R}^{m+n} : x \in A, y \in B\}$.

Problem 6

Given that $f : (0, 1) \to \mathbb{R}$ is continuous, what are the possible choices for its range $f((0, 1))$? Explain why your list exhausts all possible cases.