Department of Mathematics University of Houston

MATH 4331/6312

Introduction to Real Analysis

Fall 2017

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 6, due Thursday, October 26, 10am

Please staple this cover page to your homework. Circle your course number, 4331 or 6312. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

If a function $f : [a, b] \to \mathbb{R}$ is Lipschitz-continuous with Lipschitz constant C, then prove that for any partition P of $[a, b]$, we have $U(f, P) - L(f, P) \leq C(b - a)\text{mesh}(P)$.

Problem 2

Show that if a real-valued function f is bounded and Riemann integrable on $[a, b]$, so is $|f|, |f|(x) = |f(x)|$.

Problem 3

Show that if f and g are real-valued, bounded and Riemann integrable on $[a, b]$ and $f(x) \leq g(x)$ for each $x \in [a, b]$, then

$$\int_a^b f(x)dx \leq \int_a^b g(x)dx\,.$$

Problem 4

Prove the mean value theorem for integrals: If a real-valued function f is continuous on $[a, b]$, then there is $c \in (a, b)$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx\,.$$

Problem 5

Let $f : \mathbb{R} \to \mathbb{R}$ be continous, and fix $c > 0$. Show that the function

$$G(x) = \frac{1}{c}\int_x^{x+c} f(t)dt$$

has a continuous derivative and compute $G'(x)$.